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Monterey, California

THESIS

MATHEMATICAL MODEL

of

ARMED HELICOPTER VS. TANK DUEL

by

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Mathematical Model

of

Armed Helicopter vs. Tank Duel

by

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ABSTRACT

The purpose of this thesis is to mathematically model a duel between the armed helicopter and the tank. In addition to providing a parametric analysis of B. O. Koopman's classical Detection-Destruction Duel, two additional models were constructed and analyzed. All three models stem from stochastic versions of Lanchester's Equations but require that a unit first be detected before it is destroyed. The later two models are extensions of Koopman's model but provide for the unique capability of the helicopter to rapidly maneuver behind masking terrain, thus transitioning from the detected state back to the undetected state. With further refinement, these models may prove to be a viable alternative to the current method of computer simulation.

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I. INTRODUCTION

The objective of this thesis is to derive a simplified mathematical model of the tank vs. helicopter duel. The basic departure point in constructing this model comes from the general Detection-Destruction Duel provided by Bernard O. Koopman.[1] He extended the Lanchesterian theory by requiring that detection of a unit must precede its destruction. In other words, a unit must first detect a hostile unit before it can attempt to kill it. This general concept and Koopman's model of the Detection-Destruction Duel are presented in Chapter II.

Koopman's General Model is limited to transitions from an undetected state to a detected state only. No provisions are made to accomodate a return transition from detected back to undetected. The unique capabilities of the helicopter to rapidly hide behind hills or masking terrain, "pop down," requires that Koopman's General Model be modified to incorporate this capability of transitioning from a detected state back to the undetected state. The relative slowness of the tank and its difficulty in hiding make it unnecessary to include a similar transition for the tank. Once the tank is detected, it is assumed to remain detected even though the helicopter may be temporarily hiding or changing its location.

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[1]Koopman, B. O., "A Study of the Logical Basis of Combat Stimulation," Operations Research, Vol. 18, No. 5, p. 876.

The development of the expression for the probability state vector in Koopman's General model is straightforward, but tedious. With the addition of this return transition for the helicopter, the mathematics became considerably more unwieldy. Consequently, Model I, a simplified model, was attempted initially. Model I is presented in Chapter III. Although the results of this model are intuitively appealing, an analysis of the data curves has subsequently indicated a conceptual flaw in the model. That is, no provision was made for the tank to destroy the helicopter after the tank had been detected by the helicopter. The results produced by this model were not only insensitive to the kill rate of the helicopter; they were independent of the kill rate.

In an attempt to correct this flaw, Model II was constructed. The patterns of flow, stochastic equations, and derivation of this model are presented in Chapter IV. However, it will be obvious even to the casual observer that the end results of the equations are not quite correct. The massive mathematical reductions and bookkeeping proved too unwieldy to successfully complete.

In all three models, the probability of the tank or helicopter winning the duel is based on the Markov process of transition flows from one state to another state. The input variables are the tank and helicopter detection and kill rates and, except for the General Model, the rate at

which the tank loses detection of the helicopter. The stochastic equations resulting from these transitions provide the steady state solutions desired. In other words, the duel is allowed to continue until either the tank or helicopter wins the engagement and the results are shown as the probability of one or the other winning.

In addition to the standard Markov assumptions, additional assumptions are applicable for all three models.

--The rate at which a unit is killed remains unchanged regardless of whether the unit is detecting the opponent or has not detected the opponent. The model provides for differences in these two rates, but the parametric analysis was accomplished with the rates being equal.

--Once the tank has been detected by the helicopter, it cannot move into defalade or hide from the helicopter. This assumption is logical due to the relative slowness of the tank and the high mobility of the helicopter allowing views from various locations.

--The duel is limited to the engagement of one unit of tanks against one unit of helicopters. The engagement is terminated only when one or the other unit is completely destroyed.

II. GENERAL MODEL

A. GENERAL

The general Detection-Destruction Duel was first presented by Bernard O. Koopman.[2] The model is based on the mathematical assembly of time-dependent transition rates from one state to another. The pattern of flow, the states, and the transition rates are shown in figure 1. The states are defined as follows:

State 1: Both Helicopter and Tank undetected.

State 2: Tank detected, Helicopter undetected.

State 3: Helicopter detected, Tank undetected.

State 4: Both Helicopter and Tank detected.

State 5: Helicopter dead, Tank alive.

State 6: Tank dead, Helicopter alive.

Labels on the pattern of flow arcs represent the respective transition rates between states.

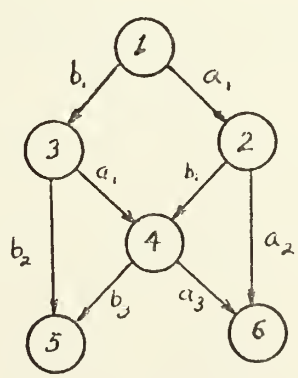

FIGURE 1

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[2] Ibid.

B. DERIVATION

By defining $P_i(t)$ as the probability that at $t > 0$, the system will be in state i, given that it was in state 1 at t=0, then the time derivative of this probability is $P_i'(t)$. The following stochastic equations can now be obtained from the flow pattern in figure 1:

$$P_1'(t) = -(a_1 + b_1)\, P_1(t) \qquad (2.1)$$

$$P_2'(t) = a_1 P_1(t) - (a_2 + b_1)\, P_2(t) \qquad (2.2)$$

$$P_3'(t) = b_1 P_1(t) - (a_1 + b_2)\, P_3(t) \qquad (2.3)$$

$$P_4'(t) = b_1 P_2(t) - (a_3 + b_3)\, P_4(t) \qquad (2.4)$$

$$P_5'(t) = b_2 P_3(t) + b_3 P_4(t) \qquad (2.5)$$

$$P_6'(t) = a_2 P_2(t) + a_3 P_4(t) \qquad (2.6)$$

These equations can be solved explicitly beginning with the first differential equation (2.1).

$$\frac{d\,P_1(t)}{dt} = P_1'(t) = -(a_1 + b_1)\, P_1(t)$$

This first order differential can be solved by separating variables as follows:

$$\int_0^t \frac{d\,P_1(t)}{P_1(t)} = -(a_1 + b_1) \int_0^t dt$$

$$\ln P_1(t) \Big|_0^t = -(a_1 + b_1)\, t \Big|_0^t$$

$$P_1(t) - P_1(0) = e^{-(a_1 + b_1)t} - 1.0$$

Since $P_1(0) = 0$,

$$P_1(t) = e^{-(a_1 + b_1)t} \qquad (2.7)$$

$P_2(t)$ can now be solved since by (2.2),

$$P_2'(t) = a_1 P_1(t) - (a_2 + b_1) P_2(t) .$$

Multiplying both sides of (2.2) by the integrating factor $e^{(a_2+b_1)t}$ and substituting the value of $P_1(t)$ from (2.7) gives the exact integral

$$(a_2 + b_1) e^{(a_2+b_1)t} P_2(t) + e^{(a_2+b_1)t} P_2'(t) = a_1 e^{-(a_1+b_1)t+(a_2+b_1)t}$$

Integrating and solving for $P_2(t)$,

$$e^{(a_2+b_1)t} P_2(t) = \left(\frac{a_1}{a_2-a_1}\right)\left(e^{(a_2+a_1)t}\right)\Big|_0^t$$

$$P_2(t) = \left(\frac{a_1}{a_2-a_1}\right)\left(e^{-(a_1+b_1)t} - e^{-(a_1+b_2)t}\right) \quad (2.8)$$

$P_3(t)$ can be solved in identical manner yielding

$$P_3(t) = \left(\frac{b_1}{b_2-b_1}\right)\left(e^{-(a_1+b_1)t} - e^{-(a_1+b_2)t}\right) \quad (2.9)$$

$P_4(t)$ can now be solved in similar manner from (2.4) although the algebraic manipulations are tedious.

$$(a_3 + b_3) P_4(t) + P_4'(t) = b_1 P_2(t) + a_1 P_3(t)$$

$$\begin{aligned} e^{(a_3+b_3)t} P_4(t) = \left(\frac{a_1 b_1}{a_2-a_1}\right) & \left[\int_0^t e^{(a_3+b_3-a_1-b_1)t} - \int_0^t e^{(a_3+b_3-a_2-b_1)t}\right] \\ + \left(\frac{a_1 b_1}{b_2-b_1}\right) & \left[\int_0^t e^{(a_3+b_3-a_1-b_1)t} - \int_0^t e^{(a_3+b_3-a_1-b_2)t}\right] \end{aligned}$$

Integrating and solving for $P_4(t)$,

$$P_4(t) = \left(\frac{a_1 b_1}{a_2 - a_1}\right)\left[\left(\frac{e^{-(a_1+b_1)t} - e^{-(a_3+b_3)t}}{a_3+b_3-a_1-b_1}\right) - \left(\frac{e^{-(a_2+b_1)t} - e^{-(a_3+b_3)t}}{a_3+b_3-a_2-b_2}\right)\right]$$

$$+\left(\frac{a_1 b_1}{b_2 - b_1}\right)\left[\left(\frac{e^{-(a_1+b_1)t} - e^{-(a_3+b_3)t}}{a_3+b_3-a_1-b_1}\right) - \left(\frac{e^{-(a_1+b_2)t} - e^{-(a_3+b_3)t}}{a_3+b_3-a_2-b_2}\right)\right]$$

Now $P_5(t)$ can be solved by integrating both sides of the fourth stochastic equation (2.5).

$$\int_0^t P_5'(t) = b_2 \int_0^t P_3(t) + b_3 \int_0^t P_4(t)$$

After integrating and allowing $t \to \infty$--for the steady state condition--

$$P_5(t\to\infty) = \left(\frac{b_1 b_2}{b_2 - b_1}\right)\left(\frac{1}{a_1+b_1} - \frac{1}{a_1+b_2}\right) + \left(\frac{a_1 b_1 b_3}{(a_2-a_1)(a_3+b_3-a_1-b_1)}\right)\left(\frac{1}{a_1+b_1} - \frac{1}{a_3+b_3}\right)$$

$$-\left(\frac{a_1 b_1 b_3}{(a_2-a_1)(a_3+b_3-a_3-b_1)}\right)\left(\frac{1}{a_2+b_1} - \frac{1}{a_3+b_3}\right)$$

$$+\left(\frac{a_1 b_1 b_3}{(b_2-b_1)(a_3+b_3-a_1-b_1)}\right)\left(\frac{1}{a_1+b_1} - \frac{1}{a_3+b_3}\right)$$

$$-\left(\frac{a_1 b_1 b_3}{(b_2-b_1)(a_3+b_3-a_1-b_2)}\right)\left(\frac{1}{a_1+b_2} - \frac{1}{a_3+b_3}\right)$$

Through straightforward, but careful bookkeeping, the steady state probability of reaching State 5 reduces to

$$P_5(t\to\infty) = \frac{b_1 b_2}{(a_1+b_1)(a_1+b_2)} + \frac{(a_1+a_2+b_1+b_2)(a_1 b_1 b_3)}{(a_1+b_1)(a_3+b_3)(a_1+b_2)(a_2+b_1)} \qquad (2.10)$$

Since the flow pattern is symetric, it follows that $P_6(t)$ can be solved in the identical manner yielding

$$P_6(t\to\infty) = \frac{a_1 a_2}{(a_1+b_1)(a_2+b_1)} + \frac{(a_1+a_2+b_1+b_2)(a_1 b_1 a_3)}{(a_1+b_1)(a_3+b_3)(a_1+b_2)(a_2+b_1)} \qquad (2.11)$$

$P_5(t \to \infty)$ and $P_6(t \to \infty)$ can be interpreted as the probabilities of victory for the tank and helicopter respectively, and their sum must add to unity.

C. PARAMETRIC ANALYSIS

The aggregation of curves with six independent variables presents a great problem in attempting to visualize sensitivity and trends. As stated previously, a_3 and b_3 are assumed to equal a_2 and b_2 respectively, thus reducing the number of different input variables to four--a_1 and b_1, the detection rates, and a_2 and b_2, the kill rates.

Figure 2 is a 3 x 3 matrix of graphs attempting to portray these remaining relationships. Each graph plots the rate at which the tank kills the helicopter (b_2) against the probability of the helicopter winning the engagement. Each individual curve represents a different rate at which the helicopter kills the tank (a_2). Within any given row of graphs, the rate at which the tank detects the helicopter (b_1) is constant. Within any given column of graphs, the rate at which the helicopter detects the tank (a_1) is constant. Along the diagonal of graphs from the upper left to the lower right corners, helicopter and tank detection rates are equal (a_1=b_1). Within this framework, relationships and sensitivities can be examined.

The first and most obvious observation that can be made concerning these graphs is that the slope of all curves is negative. That is, as the rate at which the tank kills the

helicopter (b_2) increases--other variables being held constant--the probability of the helicopter winning decreases. Correspondingly, none of the curves in any graph crosses; that is, as the rate at which the helicopter kills the tank (a_2) increases--other variables being held constant--the probability of the helicopter winning increases. Also noteworthy is the observation that as b_2 is increased, the slopes of all curves decrease at a decreasing rate. This is analogous to the observation that as a_2 is increased, the distance between the curves decreases at a decreasing rate. Thus, the concept of diminishing marginal returns, with respect to increases in the kill rates, is an apparent property of all of the curves.

Although less obvious, it can be seen that by proceeding along any row from left to right, or increasing the rate at which a helicopter detects a tank (a_1)--other variables being held constant--the probability of the helicopter winning increases at a decreasing rate and the distance between the curves on any one graph becomes greater. Correspondingly, by proceeding down any column from top to bottom, or increasing the rate at which a tank detects a helicopter (b_1)--other variables being held constant--the probability of the helicopter winning decreases at a decreasing rate and the curves become less straight. These observations imply that when the detection rate of one unit becomes much greater than that of the other, the model becomes more sensitive to

changes in the kill rate of the unit with the greater detection rate. This is best portrayed by graphs in the lower left and upper right corners of figure 2. And, similar to kill rates, the relative position of the curves indicates diminishing marginal returns with respect to increases in the detection rates (a_1 and b_1).

D. CONCLUSIONS

Two salient conclusions can be reached from the preceding analysis of the General Model.

--Firstly, the marginal returns diminish as the helicopter or tank detection and kill rates increase. Therefore, elegant, super-sophisticated, and expensive systems may not provide an adequate return on the investment.

--Secondly, the detection capability of a system should generally progress with its kill capabilities. In other words, large sums of money should not be spent achieving a high kill rate for a tank or helicopter if the detection rate is very low and vice versa. However, if the detection rate can be brought to a high level, then any increase in kill rate would have a pronounced impact on the probability of winning.

GENERAL MODEL GRAPHS

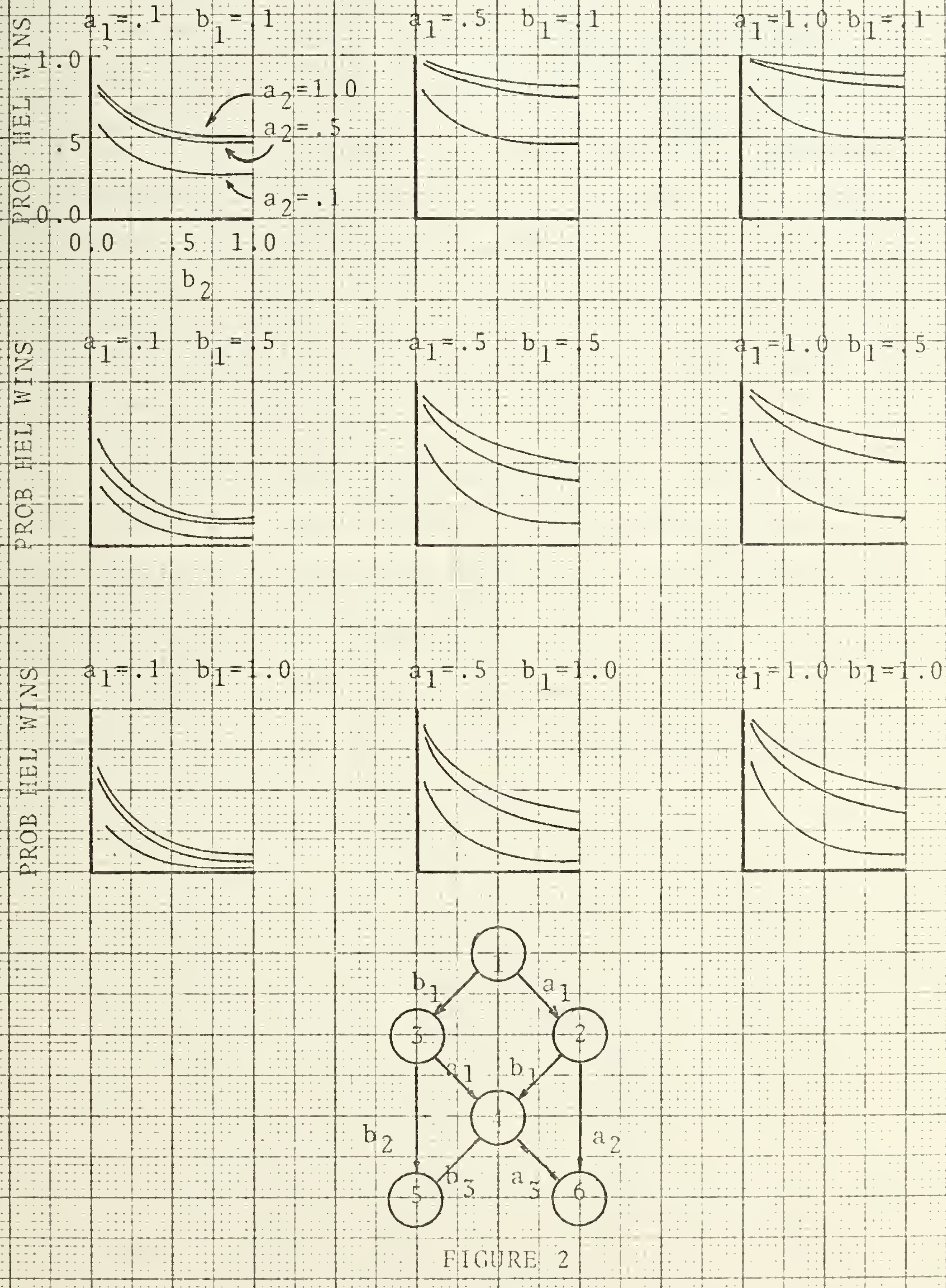

FIGURE 2

III. MODEL I

A. GENERAL

As mentioned in Chapter I, the unique capabilities of the helicopter allow for a return transition from the detected state to the undetected state. Due to the anticipated mathematical complexity in incorporating this capability into the model, a simplified model, Model I, was initially attempted. The pattern of flow, states, and transition rates are alalogous to Koopman's Detection-Destruction Model and are shown in figure 3. The states are defined as follows:

State 1: Both Helicopter and Tank undetected.

State 2: Tank detected, Helicopter undetected.

State 3: Helicopter detected, Tank undetected.

State 4: Helicopter dead, Tank alive.

State 5: Tank dead, Helicopter alive.

Labels on the pattern of flow arcs represent the respective transition rates between states.

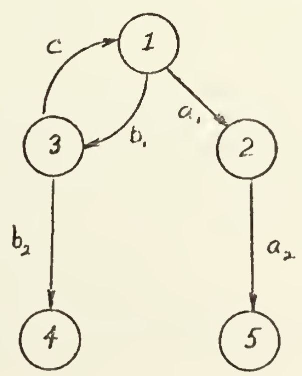

FIGURE 3

There are two notable differences between Model I and Koopman's General Model. Firstly, Model I adds the return transition from State 2 back to State 1. This modification allows the helicopter to hide behind masking terrain, out of sight from the tank. Secondly, the state allowing both the helicopter and the tank to be simultaneously detected has been eliminated to simplify the mathematics. It will be seen later that this second modification has a serious, if not disastrous impact on the usefulness of the model.

B. DERIVATION

The derivation proceeds initially much the same as the General Model, but rapidly becomes more complex. As before, the following stochastic equations can be obtained from the flow pattern in figure 3.

$$P_1'(t) = -(a_1+b_1)P_1(t) + c\,P_3(t) \qquad (3.1)$$

$$P_2'(t) = a_1 P_1(t) - a_2 P_2(t) \qquad (3.2)$$

$$P_3'(t) = b_1 P_1(t) - (b_2+c)P_3(t) \qquad (3.3)$$

$$P_4'(t) = b_2 P_3(t) \qquad (3.4)$$

$$P_5'(t) = a_2 P_2(t) \qquad (3.5)$$

The first order differential equations (3.1) and (3.3) can be solved simultaneously as follows:

Differentiating (3.1) yields

$$P_1''(t) = -(a_1+b_1)P_1'(t) + c\,P_3'(t) \qquad (3.6)$$

Now the number of simultaneous equations is equal to the number of dependent variables and a solution for $P_1(t)$ is possible.

$$P_1'(t) = -(a_1+b_1)P_1(t) + c\,P_3(t) \qquad (3.1)$$
$$P_1''(t) = -(a_1+b_1)P_1'(t) + c\,P_3'(t) \qquad (3.6)$$
$$P_3'(t) = b_1\,P_1(t) - (b_2+c)P_3(t) \qquad (3.3)$$

Solving these three equations simultaneously yields

$$P_1''(t) + (a_1+b_1+b_2+c)P_1(t) + (a_1b_2 + a_1c + b_1b_2)P_1(t) = 0 \qquad (3.7)$$

Equation (3.7) is a homogeneous linear equation of order two and can be solved by the method of the characteristic equation.

$$\lambda^2 + (a_1+b_1+b_2+c)\lambda + (a_1b_2 + a_1c + b_1b_2) = 0 \qquad (3.8)$$

From the quadratic equation

$$\lambda = \tfrac{1}{2}\left[(-a_1-b_1-b_2-c) \pm \sqrt{(a_1+b_1+b_2+c)^2 - 4(a_1b_2+a_1c+b_1b_2)}\right]$$

$$P_1(t)^+ = e^{\lambda^+ t} = e^{\frac{1}{2}\left[(-a_1-b_1-b_2-c)+\sqrt{(a_1+b_1+b_2+c)^2-4(a_1b_2+a_1c+b_1b_2)}\right]t}$$

$$P_1(t)^- = e^{\lambda^- t} = e^{\frac{1}{2}\left[(-a_1-b_1-b_2-c)-\sqrt{(a_1+b_1+b_2+c)^2-4(a_1b_2+a_1c+b_1b_2)}\right]t}$$

For simplification let $B = (-a_1-b_1-b_2-c)$

and $S = (a_1+b_1+b_2+c)^2 - 4(a_1b_2+a_1c+b_1b_2)$

Then the general solution becomes

$$P_1(t) = C'e^{\frac{1}{2}[Bt+\sqrt{S}\,t]} + D'e^{\frac{1}{2}[Bt-\sqrt{S}\,t]}$$

$$= e^{\frac{B}{2}t}\left[C'e^{\frac{\sqrt{S}}{2}t} + D'e^{-\frac{\sqrt{S}}{2}t}\right] \qquad (3.9)$$

By letting C' $= \frac{C+D}{2}$ and D' $= \frac{C-D}{2}$

equation (3.9) becomes

$$P_1(t) = e^{B/2\,t}\left[\left(\frac{C}{2}+\frac{D}{2}\right)e^{\frac{\sqrt{5}}{2}t} + \left(\frac{C}{2}-\frac{D}{2}\right)e^{-\frac{\sqrt{5}}{2}t}\right]$$

$$= e^{B/2\,t}\left[C\left(\frac{e^{\frac{\sqrt{5}}{2}t}+e^{-\frac{\sqrt{5}}{2}t}}{2}\right) + D\left(\frac{e^{\frac{\sqrt{5}}{2}t}-e^{-\frac{\sqrt{5}}{2}t}}{2}\right)\right. \quad (3.10)$$

By definition, cosh (x) $= \frac{e^{x}+e^{-x}}{2}$ and sinh (x) $= \frac{e^{x}-e^{-x}}{2}$

Therefore (3.10) becomes

$$P_1(t) = e^{B/2\,t}\left[C\cosh\frac{\sqrt{5}}{2}t + D\sinh\frac{\sqrt{5}}{2}t\right] \quad (3.11)$$

Solving for C and D, we know that since cosh (0) = 1.0 and sinh (0) = 0, then when t=0, (3.11) becomes $P_1(t=0) = C$.

But $P_1(t=0) = 1.0$ since the flow always begins in State 1 at t=0. Therefore, $P_1(t=0) = C = 1.0$.

Differentiating (3.11)

$$P_1'(t) = e^{B/2\,t}\left(C\sinh\frac{\sqrt{5}}{2}t\right)\left(\frac{\sqrt{5}}{2}\right) + e^{B/2\,t}\left(C\cosh\frac{\sqrt{5}}{2}t\right)\left(\frac{B}{2}\right)$$

$$+ e^{B/2\,t}\left(D\cosh\frac{\sqrt{5}}{2}t\right)\left(\frac{\sqrt{5}}{2}\right) + e^{B/2\,t}\left(D\sinh\frac{\sqrt{5}}{2}t\right)\left(\frac{B}{2}\right)$$

$$P_1'(t) = \frac{B}{2} + D\left(\frac{\sqrt{5}}{2}\right) \text{ when } t=0 \quad (3.12)$$

Since $P_3(t=0) = 0$, and $P_1(t=0) = 1.0$, equation (3.1) becomes

$$P_1'(t=0) = -(a_1+b_1) \quad (3.13)$$

From 3.12) and (3.13)

$$P_1'(t=0) = \frac{B}{2} + D\left(\frac{\sqrt{5}}{2}\right) = -(a_1+b_1)$$

$$D = \frac{b_2+c-a_1-b_1}{\sqrt{5}} \quad (3.14)$$

With C and D solved, equation (3.11) becomes

$$P_1(t) = e^{B/2\,t}\left(\cosh\frac{\sqrt{5}}{2}t + \frac{2R}{\sqrt{5}}\sinh\frac{\sqrt{5}}{2}t\right) \quad (3.15)$$

where $B = -(a_1 + b_1 + b_2 + c)$

$$R = \frac{b_2 + c - a_1 - b_1}{2}$$

$$S = (a_1 + b_1 + b_2 + c)^2 - 4(a_1 b_2 + a_1 c + b_1 b_2)$$

Now that the solution for $P_1(t)$ has been obtained, the remaining stochastic equations can be solved in much the same manner as Chapter II. From (3.2)

$$P_2'(t) = a_1 P_1(t) - a_2 P_2(t)$$

Multiplying both sides by the integrating factor $e^{a_2 t}$, substituting the value of $P_1(t)$ from (3.15), and then integrating gives

$$e^{a_2 t} P_2(t) = a_1 \int_0^t e^{(B/2 + a_2)t} \cosh \frac{\sqrt{S}}{2} t + \frac{a_1 2R}{\sqrt{S}} \int_0^t e^{(B/2 + a_2)t} \sinh \frac{\sqrt{S}}{2} t \tag{3.16}$$

Integrating by parts,

$$\int e^{at} \cosh bt = \frac{e^{at}}{b^2 - a^2}\left[b \sinh bt - a \cosh bt \right]$$

and

$$\int e^{at} \sinh bt = \frac{e^{at}}{b^2 - a^2}\left[b \cosh bt - a \sinh bt \right]$$

Now (3.16) can be integrated and reduced to the form

$$P_2(t) = \frac{a_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2} \left[\frac{\sqrt{S}}{2} e^{B/2\, t} \sinh \frac{\sqrt{S}}{2} t - \frac{2RT}{\sqrt{S}} e^{B/2\, t} \sinh \frac{\sqrt{S}}{2} t \right.$$

$$\left. + (R-T) e^{B/2\, t} \cosh \frac{\sqrt{S}}{2} t + (T-R) e^{-a_2 t} \right] \tag{3.17}$$

where $T = \frac{B}{2} + a_2 = \frac{2a_2 - a_1 - b_1 - b_2 - c}{2}$

In similar manner, (3.3) can be solved

$$P_3(t) = \left[\frac{b_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - R^2}\right]\left[\frac{\sqrt{S}}{2} - \frac{2R^2}{\sqrt{S}}\right]\left[e^{B/2\,t} \sinh \frac{\sqrt{S}}{2} t\right] \quad (3.18)$$

Solving (3.4) for $P_4(t)$ and letting $t \to \infty$ gives the probability of a tank victory.

$$P_4(t \to \infty) = \left[\frac{b_1 b_2}{\left(\frac{\sqrt{S}}{2}\right)^2 - R^2}\right]\left[\frac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{B}{2}\right)^2}\right]\left[R^2 - \left(\frac{\sqrt{S}}{2}\right)^2\right] = \frac{b_1 b_2}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{B}{2}\right)^2} \quad (3.19)$$

Solving (3.5) for $P_5(t)$ and letting $t \to \infty$ gives the probability of a helicopter victory.

$$P_5(t \to \infty) = \left[\frac{a_1 a_2}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{B}{2}\right)^2}\right]\left[RT - \left(\frac{\sqrt{S}}{2}\right)^2 + \frac{B}{2}(R-T)\right]$$

$$+ \left[\frac{a_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[T - R\right] \quad (3.20)$$

A recapitulation of terms follows:

$$B = -(a_1 + b_1 + b_2 + c)$$

$$R = \frac{b_2 + c - a_1 - b_1}{2}$$

$$S = (a_1 + b_1 + b_2 + c)^2 - 4(a_1 b_2 + a_1 c + b_1 b_2)$$

$$T = \frac{2a_2 - a_1 - b_1 - b_2 - c}{2}$$

C. PARAMETRIC ANALYSIS

The aggregation of curves for Model I is attemptd in a manner similar to that of the General Model. The noteable deviation, however, is that all curves are independent of the rate at which the helicopter kills the tank (a_2). The fact that the model is independent of a_2 is, of course, the major shortcoming of Model I. In words, figure 3 indicates that once the system arrives in State 2--tank detected, helicopter undetected--the system has no alternative but to proceed to State 5--helicopter wins. Consequently, the variable a_2 is removed from the analysis of Model I and replaced by c, the rate at which the helicopter transitions from the detected state back to the undetected state.

Figure 4 is a 3 x 3 matrix portraying the relationships of Model I. With the exception of c replacing a_2, the framework of figure 4 is identical to that of figure 2.

As in the General Model, by proceeding along any row from left to right, or increasing the rate at which the helicopter detects the tank (a_2), the probability of the helicopter winning increases at a decreasing rate. Correspondingly, by proceeding down any column, or increasing the rate at which the tank detects the helicopter (b_1), the probability of the helicopter winning decreases at a decreasing rate. Finally, from the appearance of all curves, an increase in the rate at which the tank kills the helicopter (b_2) decreases the probability of the helicopter

MODEL I GRAPHS

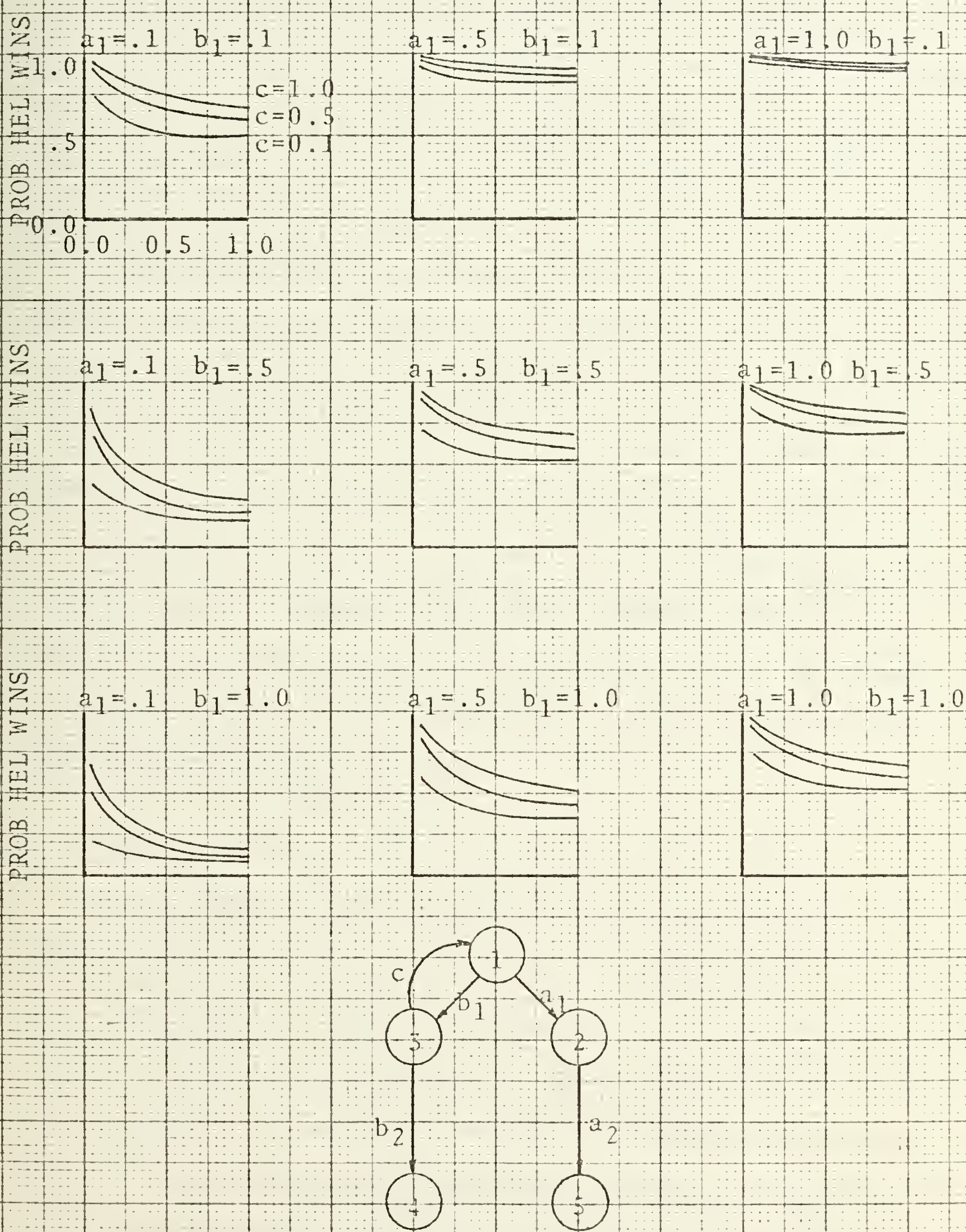

FIGURE 4

winning at a decreasing rate. Thus, the properties of diminishing marginal returns appear again with respect to increases in the detection rates of both systems (a_1 and b_1) and increases in the kill rate of the tank (b_2). Although diminishing marginal returns with respect to an increase in the "pop down" rate (c) is also indicated, in most cases it appears to be very slight.

Further observation of the "pop down" rate (c) shows Model I much more sensitive to values of c when the detection rate b_1 is greater than or equal to a_1. In other words, the "pop down" rate is a more important asset to the helicopter when the tank has the advantage in detection capability.

Also of interest in figure 4 is the observation that graphs on the diagonal from upper left to lower right are identical. This implies that when the detection capabilities are equal (a_1=b_1), the magnitude of their rates do not change the probabilities of winning.

D. CONCLUSION

Since Model I does have the serious shortcoming of being independent of the rate at which the helicopter kills the tank (a_2), care must be taken in reaching the conclusions of the model. However, in addition to those conclusions from the General Model, it appears safe to conclude that the "pop down" capability has a pronounced impact on the probability of the helicopter winning except in those cases where the detection capabilities of the helicopter are quite superior to those of the tank.

IV. MODEL II

A. GENERAL

In an attempt to eliminate the shortcoming of Model I--that the probability of winning is independent of the rate at which the helicopter can kill the tank--Model II was developed. Model II combines the proven success of Koopman's General Model with the "pop down" capability of Model I. Model II is identical to the General Model except for the addition of return transition from the detected state back to the undetected state. Thus, the shortcoming of Model I should be eliminated since Model II provides for the capability of the tank to destroy the helicopter even though the tank has been detected by the helicopter. The pattern of flow, states, and transition rates are analogous to the General Model and Model I. They are presented in figure 5 and the states are defined as follows:

State 1: Both Helicopter and Tank undetected.

State 2: Tank detected, Helicopter undetected.

State 3: Helicopter detected, Tank undetected.

State 4: Both Helicopter and Tank detected.

State 5: Helicopter dead, Tank alive.

State 6: Tank dead, Helicopter alive.

Labels on the pattern of flow arcs represent the respective transition rates between states.

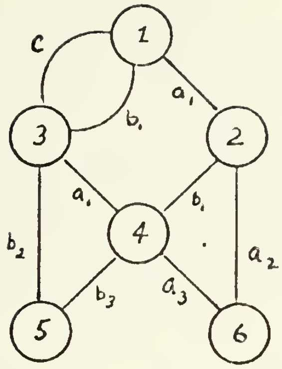

FIGURE 5

B. DERIVATION

The derivation proceeds in much the same manner as Model I; with the requirement to solve simultaneous differential equations. The procedure for deriving the model is identical to that of both the General Model and Model I, but the bookkeeping of terms is greatly increased. It is suspected that the reason the model does not provide valid results is due to errors in the accounting and transfer of complex terms. As before, the following stochastic equations can be obtained from the flow pattern in figure 5.

$$P_1'(t) = -(a_1 + b_1) P_1(t) + c P_3(t) \qquad (4.1)$$

$$P_2'(t) = a_1 P_1(t) - (a_2 + b_1) P_2(t) \qquad (4.2)$$

$$P_3'(t) = b_1 P_1(t) - (a_1 + b_2 + c) P_3(t) \qquad (4.3)$$

$$P_4'(t) = b_1 P_2(t) + a_1 P_3(t) - (a_3 + b_3) P_4(t) \qquad (4.4)$$

$$P_5'(t) = b_2 P_3(t) + b_3 P_4(t) \qquad (4.5)$$

$$P_6'(t) = a_2 P_2(t) + a_3 P_4(t) \qquad (4.6)$$

The first three of these equations are similar to those of Model I and hence $P_1(t)$, $P_2(t)$, and $P_3(t)$ can be solved procedurally the same as Model I.

Solving for three terms yields

$$P_1(t) = e^{B/2\,t}\left(\cosh \tfrac{\sqrt{S}}{2}t + \tfrac{2R}{\sqrt{S}} \sinh \tfrac{\sqrt{S}}{2}t\right) \qquad (4.7)$$

$$P_2(t) = \left[\frac{a_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{\sqrt{S}}{2} e^{B/2\,t} \sinh \tfrac{\sqrt{S}}{2}t - \tfrac{2RT}{\sqrt{S}} e^{B/2\,t} \sinh \tfrac{\sqrt{S}}{2}t\right.$$

$$\left. + (R-T)\, e^{B/2\,t} \cosh \tfrac{\sqrt{S}}{2}t + (T-R)\, e^{-(a_2+b_1)t}\right] \qquad (4.8)$$

$$P_3(t) = \left[\frac{b_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - R^2}\right]\left[\left(\frac{\sqrt{S}}{2} - \frac{2R^2}{\sqrt{S}}\right) e^{B/2\,t} \sinh \frac{\sqrt{S}}{2}\right] \qquad (4.9)$$

where, unlike Model I, $B = -(2a_1 - b_1 - b_2 - c)$

$$R = \frac{b_2 + c - b_1}{2}$$

$$T = \frac{B}{2} + a_2 + b_1 = \frac{2a_1 + 2a_2 + b_1 - b_2 - c}{2}$$

$$S = (2a_1 + b_1 + b_2 + c)^2 - 4(a_1^2 + a_1 b_1 + a_1 b_2 + a_1 c + b_1 b_2)$$

Up to this point, the equations are nearly identical to those in Model I. It is here that the bookkeeping becomes error prone. Substituting $P_2(t)$ and $P_3(t)$ into equation (4.4) and solving the differential equation for $P_4(t)$ yields

$$P_4(t) = \left[\frac{a_1 b_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{G}{2}\right)^2}\right]\left[\frac{\sqrt{S}}{2} - \frac{2RT}{\sqrt{S}}\right]\left[\frac{\sqrt{S}}{2} e^{B/2\,t} \cosh \tfrac{\sqrt{S}}{2}t - \tfrac{G}{2} e^{B/2\,t} \sinh \tfrac{\sqrt{S}}{2}t\right.$$

$$\left. - \frac{\sqrt{S}}{2} e^{-(a_3+b_3)t}\right]$$

$$+ \left[\frac{a_1 b_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{G}{2}\right)^2}\right]\left[R - T\right]\left[\frac{\sqrt{S}}{2} e^{B/2\,t} \sinh \tfrac{\sqrt{S}}{2}t - \tfrac{G}{2} e^{B/2\,t} \cosh \tfrac{\sqrt{S}}{2}t + \tfrac{G}{2} e^{-(a_3+b_3)t}\right]$$

$$+ \left[\frac{a_1 b_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{T-R}{a_3 + b_3 - a_2 - b_1}\right]\left[e^{-(a_2+b_1)t} - e^{-(a_3+b_3)t}\right]$$

$$+ \left[\frac{a_1 b_1}{\left(\frac{\sqrt{S}}{2}\right)^2 - R^2}\right]\left[\frac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{G}{2}\right)^2}\right]\left[\frac{\sqrt{S}}{2} - \frac{2R^2}{\sqrt{S}}\right]\left[\frac{\sqrt{S}}{2} e^{B/2\,t} \cosh \tfrac{\sqrt{S}}{2}t - \tfrac{G}{2} e^{B/2\,t} \sinh \tfrac{\sqrt{S}}{2}t\right.$$

$$\left. - \frac{\sqrt{S}}{2} e^{-(a_3+b_3)t}\right]$$

where $G = 2a_3 + 2b_3 - b_1 - b_2 - c$

The above, lengthy solution for $P_4(t)$ must now be substituted into equations (4.5) and (4.6). Finally, these differential equations--when integrated and $t \to \infty$ --provides steady state solutions to $P_5(t)$ and $P_6(t)$.

Although from the computer output it can readily be seen that the following solutions are not entirely correct, it may still be worthwhile to document these steady state results. It is with this guarded reservation that the following solutions to $P_5(t)$ and $P_6(t)$ are included.

$$P_5(t \to \infty) = \left[\frac{b_1 b_2}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{B}{2}\right)^2}\right] + \left[\frac{a_1 b_1 b_3}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[V1\right]\left[\left(\frac{\sqrt{S}}{2}\right)^2 - RT\right]\left[\frac{BV}{2} + \frac{GV}{2} - \frac{1}{a_3+b_3}\right]$$

$$+ \left[a_1 b_1 b_3\right]\left[V1\right]\left[\frac{BV}{2} + \frac{GV}{2} - \frac{1}{a_3+b_3}\right]$$

$$+ \left[\frac{a_1 b_1 b_3}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[V1\right]\left[R-T\right]\left[\frac{G}{2}\left(\frac{1}{a_3+b_3}\right) - \left(\frac{\sqrt{S}}{2}\right)^2(V) - \frac{GBV}{4}\right]$$

$$+ \left[\frac{a_1 b_1 b_3}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{T-R}{a_3+b_3-a_2-b_1}\right]\left[\frac{1}{a_2+b_1} - \frac{1}{a_3+b_3}\right] \quad (4.11)$$

$$P_6(t \to \infty) = \left[\frac{a_1 a_2}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[RTV - \left(\frac{\sqrt{S}}{2}\right)^2(V) + (R-T)(V)\left(\frac{B}{2}\right) + \frac{T-R}{a_2+b_1}\right]$$

$$+ \left[\frac{a_1 b_1 a_3}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[V1\right]\left[\left(\frac{\sqrt{S}}{2}\right)^2 - RT\right]\left[\frac{BV}{2} + \frac{GV}{2} - \frac{1}{a_3+b_3}\right]$$

$$+ \left[a_1 b_1 a_3\right]\left[V1\right]\left[\frac{BV}{2} + \frac{GV}{2} - \frac{1}{a_3+b_3}\right]$$

$$+ \left[\frac{a_1 b_1 a_3}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[V1\right]\left[R-T\right]\left[\frac{G}{2}\left(\frac{1}{a_3+b_3}\right) - \left(\frac{\sqrt{S}}{2}\right)^2(V) - \frac{GBV}{4}\right]$$

$$+ \left[\frac{a_1 b_1 a_3}{\left(\frac{\sqrt{S}}{2}\right)^2 - T^2}\right]\left[\frac{T-R}{a_3+b_3-a_2-b_1}\right]\left[\frac{1}{a_2+b_1} - \frac{1}{a_3+b_3}\right] \quad (4.12)$$

where $V = \dfrac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{B}{2}\right)^2}$ and $V1 = \dfrac{1}{\left(\frac{\sqrt{S}}{2}\right)^2 - \left(\frac{G}{2}\right)^2}$

C. CONCLUSIONS

In comparing the results of the General Model and Model I with those of Model II, there is reason to believe that the error in Model II is rather small and was made in transferring and substituting terms in the final differential equations. The steady state solutions of Model II have strong resemblences to those of the General Model and Model I.

Despite this, the steady state probabilities of victory for the tank and helicopter must add to unity and in no case is the probability of either allowed to exceed unity. Although not far from unity, this is clearly not the case as can be seen from the computer out from Model II.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Since Model II was not entirely successful, the conclusions for this thesis must be necessarily restricted to the General Model and Model I. An attempt to replace computer simulation data concerning tank-helicopter combat with data from these two models would indeed be premature. However, an analysis of the curves depicting the diminishing marginal returns associated with these models may provide the weapons system analyst with parametric bounds within which the most productivity can be achieved.

Concerning Model II, the concept appears sound and certainly within mathematical capability of solution. The data generated by a solution from this model may indeed compare favorably with data currently generated by computer simulation.

B. AREAS FOR FURTHER STUDY

The first priority for further study, of course, must be the correct solution of Model II. In addition, more states and transitions could be added to Model II. While this would no doubt increase the realism of the model, care must be taken to remain clear of the morass of intractable equations.

The benefits of a workable mathematical model of a tank-helicopter engagement appear to be great when compared with

the current, expensive method of computer simulation. Expansion of Lanchesterian theory and the Markov Process offers a possible alternative to this current method.

C. RECOMMENDATIONS

--That Model II be successfully completed.

--That additional research be conducted with the goal of replacing computer simulation with mathematical models in the area of tank-helicopter engagement.

--That the data from these completed mathematical models be compared with the computer simulation data and data from field tests to verify the validity of the model.

COMPUTER OUTPUT--GENERAL MODEL

PROBABILITY OF WINNING --TANK VS HELICOPTER

HELDET A1	HELKILL A2	TK DET B1	TK KILL B2	HEL WIN	TK WIN	CHECK
0.100	0.100	0.100	0.050	0.639	0.361	1.000
0.100	0.100	0.100	0.100	0.500	0.500	1.000
0.100	0.100	0.100	0.250	0.362	0.638	1.000
0.100	0.100	0.100	0.500	0.306	0.694	1.000
0.100	0.100	0.100	1.000	0.277	0.723	1.000
0.100	0.100	0.500	0.050	0.491	0.509	1.000
0.100	0.100	0.500	0.100	0.306	0.694	1.000
0.100	0.100	0.500	0.250	0.135	0.865	1.000
0.100	0.100	0.500	0.500	0.074	0.926	1.000
0.100	0.100	0.500	1.000	0.047	0.953	1.000
0.100	0.100	1.000	0.050	0.467	0.533	1.000
0.100	0.100	1.000	0.100	0.277	0.723	1.000
0.100	0.100	1.000	0.250	0.106	0.894	1.000
0.100	0.100	1.000	0.500	0.047	0.953	1.000
0.100	0.100	1.000	1.000	0.023	0.977	1.000
0.100	0.500	0.100	0.050	0.795	0.205	1.000
0.100	0.500	0.100	0.100	0.694	0.306	1.000
0.100	0.500	0.100	0.250	0.567	0.433	1.000
0.100	0.500	0.100	0.500	0.500	0.500	1.000
0.100	0.500	0.100	1.000	0.460	0.540	1.000
0.100	0.500	0.500	0.050	0.664	0.336	1.000
0.100	0.500	0.500	0.100	0.500	0.500	1.000
0.100	0.500	0.500	0.250	0.298	0.702	1.000
0.100	0.500	0.500	0.500	0.194	0.806	1.000
0.100	0.500	0.500	1.000	0.136	0.864	1.000
0.100	0.500	1.000	0.050	0.636	0.364	1.000
0.100	0.500	1.000	0.100	0.460	0.540	1.000
0.100	0.500	1.000	0.250	0.244	0.756	1.000
0.100	0.500	1.000	0.500	0.136	0.864	1.000
0.100	0.500	1.000	1.000	0.078	0.922	1.000
0.100	1.000	0.100	0.050	0.815	0.185	1.000
0.100	1.000	0.100	0.100	0.723	0.277	1.000
0.100	1.000	0.100	0.250	0.605	0.395	1.000
0.100	1.000	0.100	0.500	0.540	0.460	1.000
0.100	1.000	0.100	1.000	0.500	0.500	1.000
0.100	1.000	0.500	0.050	0.693	0.307	1.000
0.100	1.000	0.500	0.100	0.540	0.460	1.000
0.100	1.000	0.500	0.250	0.346	0.654	1.000
0.100	1.000	0.500	0.500	0.241	0.759	1.000
0.100	1.000	0.500	1.000	0.177	0.823	1.000
0.100	1.000	1.000	0.050	0.666	0.334	1.000
0.100	1.000	1.000	0.100	0.500	0.500	1.000
0.100	1.000	1.000	0.250	0.290	0.710	1.000
0.100	1.000	1.000	0.500	0.177	0.823	1.000
0.100	1.000	1.000	1.000	0.110	0.890	1.000

HELDET	HELKILL	TK DET	TK KILL	HEL WIN	TK WIN	CHECK
0.500	0.100	0.100	0.050	0.795	0.205	1.000
0.500	0.100	0.100	0.100	0.694	0.306	1.000
0.500	0.100	0.100	0.250	0.567	0.433	1.000
0.500	0.100	0.100	0.500	0.500	0.500	1.000
0.500	0.100	0.100	1.000	0.460	0.540	1.000
0.500	0.100	0.500	0.050	0.664	0.336	1.000
0.500	0.100	0.500	0.100	0.500	0.500	1.000
0.500	0.100	0.500	0.250	0.298	0.702	1.000
0.500	0.100	0.500	0.500	0.194	0.806	1.000
0.500	0.100	0.500	1.000	0.136	0.864	1.000
0.500	0.100	1.000	0.050	0.636	0.364	1.000
0.500	0.100	1.000	0.100	0.460	0.540	1.000
0.500	0.100	1.000	0.250	0.244	0.756	1.000
0.500	0.100	1.000	0.500	0.136	0.864	1.000
0.500	0.100	1.000	1.000	0.078	0.922	1.000
0.500	0.500	0.100	0.050	0.958	0.042	1.000
0.500	0.500	0.100	0.100	0.926	0.074	1.000
0.500	0.500	0.100	0.250	0.861	0.139	1.000
0.500	0.500	0.100	0.500	0.806	0.194	1.000
0.500	0.500	0.100	1.000	0.759	0.241	1.000
0.500	0.500	0.500	0.050	0.890	0.110	1.000
0.500	0.500	0.500	0.100	0.806	0.194	1.000
0.500	0.500	0.500	0.250	0.639	0.361	1.000
0.500	0.500	0.500	0.500	0.500	0.500	1.000
0.500	0.500	0.500	1.000	0.389	0.611	1.000
0.500	0.500	1.000	0.050	0.864	0.136	1.000
0.500	0.500	1.000	0.100	0.759	0.241	1.000
0.500	0.500	1.000	0.250	0.556	0.444	1.000
0.500	0.500	1.000	0.500	0.389	0.611	1.000
0.500	0.500	1.000	1.000	0.259	0.741	1.000
0.500	1.000	0.100	0.050	0.974	0.026	1.000
0.500	1.000	0.100	0.100	0.953	0.047	1.000
0.500	1.000	0.100	0.250	0.907	0.093	1.000
0.500	1.000	0.100	0.500	0.864	0.136	1.000
0.500	1.000	0.100	1.000	0.823	0.177	1.000
0.500	1.000	0.500	0.050	0.925	0.075	1.000
0.500	1.000	0.500	0.100	0.864	0.136	1.000
0.500	1.000	0.500	0.250	0.733	0.267	1.000
0.500	1.000	0.500	0.500	0.611	0.389	1.000
0.500	1.000	0.500	1.000	0.500	0.500	1.000
0.500	1.000	1.000	0.050	0.903	0.097	1.000
0.500	1.000	1.000	0.100	0.823	0.177	1.000
0.500	1.000	1.000	0.250	0.656	0.344	1.000
0.500	1.000	1.000	0.500	0.500	0.500	1.000
0.500	1.000	1.000	1.000	0.361	0.639	1.000
1.000	0.100	0.100	0.050	0.815	0.185	1.000

HELDET	HELKILL	TK DET	TK KILL	HEL WIN	TK WIN	CHECK
1.000	0.100	0.100	0.100	0.723	0.277	1.000
1.000	0.100	0.100	0.250	0.605	0.395	1.000
1.000	0.100	0.100	0.500	0.540	0.460	1.000
1.000	0.100	0.100	1.000	0.500	0.500	1.000
1.000	0.100	0.500	0.050	0.693	0.307	1.000
1.000	0.100	0.500	0.100	0.540	0.460	1.000
1.000	0.100	0.500	0.250	0.346	0.654	1.000
1.000	0.100	0.500	0.500	0.241	0.759	1.000
1.000	0.100	0.500	1.000	0.177	0.823	1.000
1.000	0.100	1.000	0.050	0.666	0.334	1.000
1.000	0.100	1.000	0.100	0.500	0.500	1.000
1.000	0.100	1.000	0.250	0.290	0.710	1.000
1.000	0.100	1.000	0.500	0.177	0.823	1.000
1.000	0.100	1.000	1.000	0.110	0.890	1.000
1.000	0.500	0.100	0.050	0.974	0.026	1.000
1.000	0.500	0.100	0.100	0.953	0.047	1.000
1.000	0.500	0.100	0.250	0.907	0.093	1.000
1.000	0.500	0.100	0.500	0.864	0.136	1.000
1.000	0.500	0.100	1.000	0.823	0.177	1.000
1.000	0.500	0.500	0.050	0.925	0.075	1.000
1.000	0.500	0.500	0.100	0.864	0.136	1.000
1.000	0.500	0.500	0.250	0.733	0.267	1.000
1.000	0.500	0.500	0.500	0.611	0.389	1.000
1.000	0.500	0.500	1.000	0.500	0.500	1.000
1.000	0.500	1.000	0.050	0.903	0.097	1.000
1.000	0.500	1.000	0.100	0.823	0.177	1.000
1.000	0.500	1.000	0.250	0.656	0.344	1.000
1.000	0.500	1.000	0.500	0.500	0.500	1.000
1.000	0.500	1.000	1.000	0.361	0.639	1.000
1.000	1.000	0.100	0.050	0.988	0.012	1.000
1.000	1.000	0.100	0.100	0.977	0.023	1.000
1.000	1.000	0.100	0.250	0.951	0.049	1.000
1.000	1.000	0.100	0.500	0.922	0.078	1.000
1.000	1.000	0.100	1.000	0.890	0.110	1.000
1.000	1.000	0.500	0.050	0.958	0.042	1.000
1.000	1.000	0.500	0.100	0.922	0.078	1.000
1.000	1.000	0.500	0.250	0.836	0.164	1.000
1.000	1.000	0.500	0.500	0.741	0.259	1.000
1.000	1.000	0.500	1.000	0.639	0.361	1.000
1.000	1.000	1.000	0.050	0.942	0.058	1.000
1.000	1.000	1.000	0.100	0.890	0.110	1.000
1.000	1.000	1.000	0.250	0.770	0.230	1.000
1.000	1.000	1.000	0.500	0.639	0.361	1.000
1.000	1.000	1.000	1.000	0.500	0.500	1.000

COMPUTER OUTPUT -- MODEL I

PROBABILITY OF WINNING -- TANK VS HELICOPTER

TK DET A1	TK KILL A2	HEL DET B1	HEL KIL B2	HEL HID C	TK WIN	HEL WIN	CHECK
0.100	0.100	0.100	0.050	0.100	0.250	0.750	1.000
0.100	0.100	0.100	0.050	0.500	0.083	0.917	1.000
0.100	0.100	0.100	0.050	1.000	0.045	0.955	1.000
0.100	0.100	0.100	0.100	0.500	0.143	0.375	0.518
0.100	0.100	0.100	0.100	1.000	0.083	1.000	1.083
0.100	0.100	0.100	0.250	0.100	0.417	0.583	1.000
0.100	0.100	0.100	0.250	0.500	0.250	0.750	1.000
0.100	0.100	0.100	0.250	1.000	0.167	0.833	1.000
0.100	0.100	0.100	0.500	0.100	0.455	0.545	1.000
0.100	0.100	0.100	0.500	0.500	0.333	0.667	1.000
0.100	0.100	0.100	0.500	1.000	0.250	0.750	1.000
0.100	0.100	0.100	1.000	0.100	0.476	0.524	1.000
0.100	0.100	0.100	1.000	0.500	0.400	0.600	1.000
0.100	0.100	0.100	1.000	1.000	0.333	0.667	1.000
0.100	0.100	0.500	0.050	0.100	0.625	0.375	1.000
0.100	0.100	0.500	0.050	0.500	0.312	0.688	1.000
0.100	0.100	0.500	0.050	1.000	0.192	0.808	1.000
0.100	0.100	0.500	0.100	0.100	0.714	0.0	0.714
0.100	0.100	0.500	0.100	0.500	0.455	0.375	0.830
0.100	0.100	0.500	0.100	1.000	0.312	0.688	1.000
0.100	0.100	0.500	0.250	0.100	0.781	0.219	1.000
0.100	0.100	0.500	0.250	0.500	0.625	0.375	1.000
0.100	0.100	0.500	0.250	1.000	0.500	0.500	1.000
0.100	0.100	0.500	0.500	0.100	0.806	0.194	1.000
0.100	0.100	0.500	0.500	0.500	0.714	0.286	1.000
0.100	0.100	0.500	0.500	1.000	0.625	0.375	1.000
0.100	0.100	0.500	1.000	0.100	0.820	0.180	1.000
0.100	0.100	0.500	1.000	0.500	0.769	0.231	1.000
0.100	0.100	0.500	1.000	1.000	0.714	0.286	1.000
0.100	0.100	1.000	0.050	0.100	0.769	0.231	1.000
0.100	0.100	1.000	0.050	0.500	0.476	0.524	1.000
0.100	0.100	1.000	0.050	1.000	0.323	0.677	1.000
0.100	0.100	1.000	0.100	0.100	0.833	0.137	0.970
0.100	0.100	1.000	0.100	0.500	0.625	0.180	0.805
0.100	0.100	1.000	0.100	1.000	0.476	0.145	0.621
0.100	0.100	1.000	0.250	0.100	0.877	0.123	1.000
0.100	0.100	1.000	0.250	0.500	0.769	0.231	1.000
0.100	0.100	1.000	0.250	1.000	0.667	0.333	1.000
0.100	0.100	1.000	0.500	0.100	0.893	0.107	1.000
0.100	0.100	1.000	0.500	0.500	0.833	0.167	1.000
0.100	0.100	1.000	0.500	1.000	0.769	0.231	1.000
0.100	0.100	1.000	1.000	0.100	0.901	0.099	1.000
0.100	0.100	1.000	1.000	0.500	0.870	0.130	1.000
0.100	0.100	1.000	1.000	1.000	0.833	0.167	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.100	0.500	0.100	0.050	0.100	0.250	0.750	1.000
0.100	0.500	0.100	0.050	0.500	0.083	0.917	1.000
0.100	0.500	0.100	0.050	1.000	0.045	0.955	1.000
0.100	0.500	0.100	0.100	0.100	0.333	0.667	1.000
0.100	0.500	0.100	0.100	0.500	0.143	0.857	1.000
0.100	0.500	0.100	0.100	1.000	0.083	0.917	1.000
0.100	0.500	0.100	0.250	0.100	0.417	0.583	1.000
0.100	0.500	0.100	0.250	0.500	0.250	0.750	1.000
0.100	0.500	0.100	0.250	1.000	0.167	0.833	1.000
0.100	0.500	0.100	0.500	0.100	0.455	0.545	1.000
0.100	0.500	0.100	0.500	0.500	0.333	0.667	1.000
0.100	0.500	0.100	0.500	1.000	0.250	0.750	1.000
0.100	0.500	0.100	1.000	0.100	0.476	0.524	1.000
0.100	0.500	0.100	1.000	0.500	0.400	0.600	1.000
0.100	0.500	0.100	1.000	1.000	0.333	0.667	1.000
0.100	0.500	0.500	0.050	0.100	0.625	0.375	1.000
0.100	0.500	0.500	0.050	0.500	0.312	0.687	1.000
0.100	0.500	0.500	0.050	1.000	0.192	0.808	1.000
0.100	0.500	0.500	0.100	0.100	0.714	0.286	1.000
0.100	0.500	0.500	0.100	0.500	0.455	0.545	1.000
0.100	0.500	0.500	0.100	1.000	0.312	0.687	1.000
0.100	0.500	0.500	0.250	0.100	0.781	0.219	1.000
0.100	0.500	0.500	0.250	0.500	0.625	0.375	1.000
0.100	0.500	0.500	0.250	1.000	0.500	0.500	1.000
0.100	0.500	0.500	0.500	0.100	0.806	0.194	1.000
0.100	0.500	0.500	0.500	0.500	0.714	0.286	1.000
0.100	0.500	0.500	0.500	1.000	0.625	0.375	1.000
0.100	0.500	0.500	1.000	0.100	0.820	0.180	1.000
0.100	0.500	0.500	1.000	0.500	0.769	0.231	1.000
0.100	0.500	0.500	1.000	1.000	0.714	0.286	1.000
0.100	0.500	1.000	0.050	0.100	0.769	0.231	1.000
0.100	0.500	1.000	0.050	0.500	0.476	0.524	1.000
0.100	0.500	1.000	0.050	1.000	0.323	0.677	1.000
0.100	0.500	1.000	0.100	0.100	0.833	0.167	1.000
0.100	0.500	1.000	0.100	0.500	0.625	0.375	1.000
0.100	0.500	1.000	0.100	1.000	0.476	0.524	1.000
0.100	0.500	1.000	0.250	0.100	0.877	0.123	1.000
0.100	0.500	1.000	0.250	0.500	0.769	0.231	1.000
0.100	0.500	1.000	0.250	1.000	0.667	0.333	1.000
0.100	0.500	1.000	0.500	0.100	0.893	0.107	1.000
0.100	0.500	1.000	0.500	0.500	0.833	0.167	1.000
0.100	0.500	1.000	0.500	1.000	0.769	0.231	1.000
0.100	0.500	1.000	1.000	0.100	0.901	0.099	1.000
0.100	0.500	1.000	1.000	0.500	0.870	0.130	1.000
0.100	0.500	1.000	1.000	1.000	0.833	0.167	1.000
0.100	1.000	0.100	0.050	0.100	0.250	0.750	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.100	1.000	0.100	0.050	0.500	0.083	0.917	1.000
0.100	1.000	0.100	0.050	1.000	0.045	0.955	1.000
0.100	1.000	0.100	0.100	0.100	0.333	0.667	1.000
0.100	1.000	0.100	0.100	0.500	0.143	0.857	1.000
0.100	1.000	0.100	0.100	1.000	0.083	0.917	1.000
0.100	1.000	0.100	0.250	0.100	0.417	0.583	1.000
0.100	1.000	0.100	0.250	0.500	0.250	0.750	1.000
0.100	1.000	0.100	0.250	1.000	0.167	0.833	1.000
0.100	1.000	0.100	0.500	0.100	0.455	0.545	1.000
0.100	1.000	0.100	0.500	0.500	0.333	0.667	1.000
0.100	1.000	0.100	0.500	1.000	0.250	0.750	1.000
0.100	1.000	0.100	1.000	0.100	0.476	0.524	1.000
0.100	1.000	0.100	1.000	0.500	0.400	0.600	1.000
0.100	1.000	0.100	1.000	1.000	0.333	0.667	1.000
0.100	1.000	0.500	0.050	0.100	0.625	0.375	1.000
0.100	1.000	0.500	0.050	0.500	0.312	0.687	1.000
0.100	1.000	0.500	0.050	1.000	0.192	0.808	1.000
0.100	1.000	0.500	0.100	0.100	0.714	0.286	1.000
0.100	1.000	0.500	0.100	0.500	0.455	0.545	1.000
0.100	1.000	0.500	0.100	1.000	0.312	0.687	1.000
0.100	1.000	0.500	0.250	0.100	0.781	0.219	1.000
0.100	1.000	0.500	0.250	0.500	0.625	0.375	1.000
0.100	1.000	0.500	0.250	1.000	0.500	0.500	1.000
0.100	1.000	0.500	0.500	0.100	0.806	0.194	1.000
0.100	1.000	0.500	0.500	0.500	0.714	0.286	1.000
0.100	1.000	0.500	0.500	1.000	0.625	0.375	1.000
0.100	1.000	0.500	1.000	0.100	0.820	0.180	1.000
0.100	1.000	0.500	1.000	0.500	0.769	0.231	1.000
0.100	1.000	0.500	1.000	1.000	0.714	0.286	1.000
0.100	1.000	1.000	0.050	0.100	0.769	0.231	1.000
0.100	1.000	1.000	0.050	0.500	0.476	0.524	1.000
0.100	1.000	1.000	0.050	1.000	0.323	0.677	1.000
0.100	1.000	1.000	0.100	0.100	0.833	0.167	1.000
0.100	1.000	1.000	0.100	0.500	0.625	0.375	1.000
0.100	1.000	1.000	0.100	1.000	0.476	0.524	1.000
0.100	1.000	1.000	0.250	0.100	0.877	0.123	1.000
0.100	1.000	1.000	0.250	0.500	0.769	0.231	1.000
0.100	1.000	1.000	0.250	1.000	0.667	0.333	1.000
0.100	1.000	1.000	0.500	0.100	0.893	0.107	1.000
0.100	1.000	1.000	0.500	0.500	0.833	0.167	1.000
0.100	1.000	1.000	0.500	1.000	0.769	0.231	1.000
0.100	1.000	1.000	1.000	0.100	0.901	0.099	1.000
0.100	1.000	1.000	1.000	0.500	0.870	0.130	1.000
0.100	1.000	1.000	1.000	1.000	0.833	0.167	1.000
0.500	0.100	0.100	0.050	0.100	0.062	0.938	1.000
0.500	0.100	0.100	0.050	0.500	0.018	0.982	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.500	0.100	0.100	0.050	1.000	0.009	0.991	1.000
0.500	0.100	0.100	0.100	0.100	0.091	0.909	1.000
0.500	0.100	0.100	0.100	0.500	0.032	0.968	1.000
0.500	0.100	0.100	0.100	1.000	0.018	0.982	1.000
0.500	0.100	0.100	0.250	0.100	0.125	0.875	1.000
0.500	0.100	0.100	0.250	0.500	0.062	0.938	1.000
0.500	0.100	0.100	0.250	1.000	0.038	0.962	1.000
0.500	0.100	0.100	0.500	0.100	0.143	0.857	1.000
0.500	0.100	0.100	0.500	0.500	0.091	0.909	1.000
0.500	0.100	0.100	0.500	1.000	0.063	0.937	1.000
0.500	0.100	0.100	1.000	0.100	0.154	0.846	1.000
0.500	0.100	0.100	1.000	0.500	0.118	0.882	1.000
0.500	0.100	0.100	1.000	1.000	0.091	0.909	1.000
0.500	0.100	0.500	0.050	0.100	0.250	0.750	1.000
0.500	0.100	0.500	0.050	0.500	0.083	0.917	1.000
0.500	0.100	0.500	0.050	1.000	0.045	0.955	1.000
0.500	0.100	0.500	0.100	0.100	0.333	0.667	1.000
0.500	0.100	0.500	0.100	0.500	0.143	0.857	1.000
0.500	0.100	0.500	0.100	1.000	0.083	0.917	1.000
0.500	0.100	0.500	0.250	0.100	0.417	0.583	1.000
0.500	0.100	0.500	0.250	0.500	0.250	0.750	1.000
0.500	0.100	0.500	0.250	1.000	0.167	0.833	1.000
0.500	0.100	0.500	0.500	0.100	0.455	0.545	1.000
0.500	0.100	0.500	0.500	0.500	0.333	0.667	1.000
0.500	0.100	0.500	0.500	1.000	0.250	0.750	1.000
0.500	0.100	0.500	1.000	0.100	0.476	0.524	1.000
0.500	0.100	0.500	1.000	0.500	0.400	0.600	1.000
0.500	0.100	0.500	1.000	1.000	0.333	0.667	1.000
0.500	0.100	1.000	0.050	0.100	0.400	0.600	1.000
0.500	0.100	1.000	0.050	0.500	0.154	0.846	1.000
0.500	0.100	1.000	0.050	1.000	0.087	0.913	1.000
0.500	0.100	1.000	0.100	0.100	0.500	0.500	1.000
0.500	0.100	1.000	0.100	0.500	0.250	0.750	1.000
0.500	0.100	1.000	0.100	1.000	0.154	0.846	1.000
0.500	0.100	1.000	0.250	0.100	0.588	0.412	1.000
0.500	0.100	1.000	0.250	0.500	0.400	0.600	1.000
0.500	0.100	1.000	0.250	1.000	0.286	0.714	1.000
0.500	0.100	1.000	0.500	0.100	0.625	0.375	1.000
0.500	0.100	1.000	0.500	0.500	0.500	0.500	1.000
0.500	0.100	1.000	0.500	1.000	0.400	0.600	1.000
0.500	0.100	1.000	1.000	0.100	0.645	0.355	1.000
0.500	0.100	1.000	1.000	0.500	0.571	0.429	1.000
0.500	0.100	1.000	1.000	1.000	0.500	0.500	1.000
0.500	0.500	0.100	0.050	0.100	0.062	0.938	1.000
0.500	0.500	0.100	0.050	0.500	0.018	0.982	1.000
0.500	0.500	0.100	0.050	1.000	0.009	0.991	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.500	0.500	0.100	0.100	0.100	0.091	0.909	1.000
0.500	0.500	0.100	0.100	0.500	0.032	0.968	1.000
0.500	0.500	0.100	0.100	1.000	0.018	0.982	1.000
0.500	0.500	0.100	0.250	0.100	0.125	0.875	1.000
0.500	0.500	0.100	0.250	0.500	0.062	0.938	1.000
0.500	0.500	0.100	0.250	1.000	0.038	0.962	1.000
0.500	0.500	0.100	0.500	0.100	0.143	0.813	0.955
0.500	0.500	0.100	0.500	0.500	0.091	1.000	1.091
0.500	0.500	0.100	0.500	1.000	0.063	1.313	1.375
0.500	0.500	0.100	1.000	0.100	0.154	0.846	1.000
0.500	0.500	0.100	1.000	0.500	0.118	0.882	1.000
0.500	0.500	0.100	1.000	1.000	0.091	0.909	1.000
0.500	0.500	0.500	0.050	0.100	0.250	0.750	1.000
0.500	0.500	0.500	0.050	0.500	0.083	0.917	1.000
0.500	0.500	0.500	0.050	1.000	0.045	0.955	1.000
0.500	0.500	0.500	0.100	0.100	0.333	0.667	1.000
0.500	0.500	0.500	0.100	0.500	0.143	0.857	1.000
0.500	0.500	0.500	0.100	1.000	0.083	0.917	1.000
0.500	0.500	0.500	0.250	0.100	0.417	0.583	1.000
0.500	0.500	0.500	0.250	0.500	0.250	0.750	1.000
0.500	0.500	0.500	0.250	1.000	0.167	0.833	1.000
0.500	0.500	0.500	0.500	0.100	0.455	0.625	1.080
0.500	0.500	0.500	1.000	0.100	0.476	0.524	1.000
0.500	0.500	0.500	1.000	0.500	0.400	0.600	1.000
0.500	0.500	0.500	1.000	1.000	0.333	0.667	1.000
0.500	0.500	1.000	0.050	0.100	0.400	0.600	1.000
0.500	0.500	1.000	0.050	0.500	0.154	0.846	1.000
0.500	0.500	1.000	0.050	1.000	0.087	0.913	1.000
0.500	0.500	1.000	0.100	0.100	0.500	0.500	1.000
0.500	0.500	1.000	0.100	0.500	0.250	0.750	1.000
0.500	0.500	1.000	0.100	1.000	0.154	0.846	1.000
0.500	0.500	1.000	0.250	0.100	0.588	0.412	1.000
0.500	0.500	1.000	0.250	0.500	0.400	0.600	1.000
0.500	0.500	1.000	0.250	1.000	0.286	0.714	1.000
0.500	0.500	1.000	0.500	0.100	0.625	0.563	1.188
0.500	0.500	1.000	1.000	0.100	0.645	0.355	1.000
0.500	0.500	1.000	1.000	0.500	0.571	0.429	1.000
0.500	0.500	1.000	1.000	1.000	0.500	0.500	1.000
0.500	1.000	0.100	0.050	0.100	0.062	0.937	1.000
0.500	1.000	0.100	0.050	0.500	0.018	0.982	1.000
0.500	1.000	0.100	0.050	1.000	0.009	0.991	1.000
0.500	1.000	0.100	0.100	0.100	0.091	0.909	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.500	1.000	0.100	0.100	0.500	0.032	0.968	1.000
0.500	1.000	0.100	0.100	1.000	0.018	0.982	1.000
0.500	1.000	0.100	0.250	0.100	0.125	0.875	1.000
0.500	1.000	0.100	0.250	0.500	0.062	0.937	1.000
0.500	1.000	0.100	0.250	1.000	0.038	0.962	1.000
0.500	1.000	0.100	0.500	0.100	0.143	0.857	1.000
0.500	1.000	0.100	0.500	0.500	0.091	0.909	1.000
0.500	1.000	0.100	0.500	1.000	0.063	0.937	1.000
0.500	1.000	0.100	1.000	0.100	0.154	0.846	1.000
0.500	1.000	0.100	1.000	0.500	0.118	0.882	1.000
0.500	1.000	0.100	1.000	1.000	0.091	0.909	1.000
0.500	1.000	0.500	0.050	0.100	0.250	0.750	1.000
0.500	1.000	0.500	0.050	0.500	0.083	0.917	1.000
0.500	1.000	0.500	0.050	1.000	0.045	0.955	1.000
0.500	1.000	0.500	0.100	0.100	0.333	0.667	1.000
0.500	1.000	0.500	0.100	0.500	0.143	0.857	1.000
0.500	1.000	0.500	0.100	1.000	0.083	0.917	1.000
0.500	1.000	0.500	0.250	0.100	0.417	0.583	1.000
0.500	1.000	0.500	0.250	0.500	0.250	0.750	1.000
0.500	1.000	0.500	0.250	1.000	0.167	0.833	1.000
0.500	1.000	0.500	0.500	0.100	0.455	0.545	1.000
0.500	1.000	0.500	0.500	0.500	0.333	0.667	1.000
0.500	1.000	0.500	0.500	1.000	0.250	0.750	1.000
0.500	1.000	0.500	1.000	0.100	0.476	0.524	1.000
0.500	1.000	0.500	1.000	0.500	0.400	0.600	1.000
0.500	1.000	0.500	1.000	1.000	0.333	0.667	1.000
0.500	1.000	1.000	0.050	0.100	0.400	0.600	1.000
0.500	1.000	1.000	0.050	0.500	0.154	0.846	1.000
0.500	1.000	1.000	0.050	1.000	0.087	0.913	1.000
0.500	1.000	1.000	0.100	0.100	0.500	0.500	1.000
0.500	1.000	1.000	0.100	0.500	0.250	0.750	1.000
0.500	1.000	1.000	0.100	1.000	0.154	0.846	1.000
0.500	1.000	1.000	0.250	0.100	0.588	0.412	1.000
0.500	1.000	1.000	0.250	0.500	0.400	0.600	1.000
0.500	1.000	1.000	0.250	1.000	0.286	0.714	1.000
0.500	1.000	1.000	0.500	0.100	0.625	0.375	1.000
0.500	1.000	1.000	0.500	0.500	0.500	0.500	1.000
0.500	1.000	1.000	0.500	1.000	0.400	0.600	1.000
0.500	1.000	1.000	1.000	0.100	0.645	0.355	1.000
0.500	1.000	1.000	1.000	0.500	0.571	0.429	1.000
0.500	1.000	1.000	1.000	1.000	0.500	0.500	1.000
1.000	0.100	0.100	0.050	0.100	0.032	0.968	1.000
1.000	0.100	0.100	0.050	0.500	0.009	0.991	1.000
1.000	0.100	0.100	0.050	1.000	0.005	0.995	1.000
1.000	0.100	0.100	0.100	0.100	0.048	0.952	1.000
1.000	0.100	0.100	0.100	0.500	0.016	0.984	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
1.000	0.100	0.100	0.100	1.000	0.009	0.991	1.000
1.000	0.100	0.100	0.250	0.100	0.067	0.933	1.000
1.000	0.100	0.100	0.250	0.500	0.032	0.968	1.000
1.000	0.100	0.100	0.250	1.000	0.020	0.980	1.000
1.000	0.100	0.100	0.500	0.100	0.077	0.923	1.000
1.000	0.100	0.100	0.500	0.500	0.048	0.952	1.000
1.000	0.100	0.100	0.500	1.000	0.032	0.968	1.000
1.000	0.100	0.100	1.000	0.100	0.083	0.917	1.000
1.000	0.100	0.100	1.000	0.500	0.063	0.938	1.000
1.000	0.100	0.100	1.000	1.000	0.048	0.952	1.000
1.000	0.100	0.500	0.050	0.100	0.143	0.857	1.000
1.000	0.100	0.500	0.050	0.500	0.043	0.957	1.000
1.000	0.100	0.500	0.050	1.000	0.023	0.977	1.000
1.000	0.100	0.500	0.100	0.100	0.200	0.800	1.000
1.000	0.100	0.500	0.100	0.500	0.077	0.923	1.000
1.000	0.100	0.500	0.100	1.000	0.043	0.957	1.000
1.000	0.100	0.500	0.250	0.100	0.263	0.737	1.000
1.000	0.100	0.500	0.250	0.500	0.143	0.857	1.000
1.000	0.100	0.500	0.250	1.000	0.091	0.909	1.000
1.000	0.100	0.500	0.500	0.100	0.294	0.706	1.000
1.000	0.100	0.500	0.500	0.500	0.200	0.800	1.000
1.000	0.100	0.500	0.500	1.000	0.143	0.857	1.000
1.000	0.100	0.500	1.000	0.100	0.313	0.688	1.000
1.000	0.100	0.500	1.000	0.500	0.250	0.750	1.000
1.000	0.100	0.500	1.000	1.000	0.200	0.800	1.000
1.000	0.100	1.000	0.050	0.100	0.250	0.750	1.000
1.000	0.100	1.000	0.050	0.500	0.083	0.917	1.000
1.000	0.100	1.000	0.050	1.000	0.045	0.955	1.000
1.000	0.100	1.000	0.100	0.100	0.333	0.667	1.000
1.000	0.100	1.000	0.100	0.500	0.143	0.857	1.000
1.000	0.100	1.000	0.100	1.000	0.083	0.917	1.000
1.000	0.100	1.000	0.250	0.100	0.417	0.583	1.000
1.000	0.100	1.000	0.250	0.500	0.250	0.750	1.000
1.000	0.100	1.000	0.250	1.000	0.167	0.833	1.000
1.000	0.100	1.000	0.500	0.100	0.455	0.545	1.000
1.000	0.100	1.000	0.500	0.500	0.333	0.667	1.000
1.000	0.100	1.000	0.500	1.000	0.250	0.750	1.000
1.000	0.100	1.000	1.000	0.100	0.476	0.524	1.000
1.000	0.100	1.000	1.000	0.500	0.400	0.600	1.000
1.000	0.100	1.000	1.000	1.000	0.333	0.667	1.000
1.000	0.500	0.100	0.050	0.100	0.032	0.968	1.000
1.000	0.500	0.100	0.050	0.500	0.009	0.991	1.000
1.000	0.500	0.100	0.050	1.000	0.005	0.995	1.000
1.000	0.500	0.100	0.100	0.100	0.048	0.952	1.000
1.000	0.500	0.100	0.100	0.500	0.016	0.984	1.000
1.000	0.500	0.100	0.100	1.000	0.009	0.991	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
1.000	0.500	0.100	0.250	0.100	0.067	0.933	1.000
1.000	0.500	0.100	0.250	0.500	0.032	0.968	1.000
1.000	0.500	0.100	0.250	1.000	0.020	0.980	1.000
1.000	0.500	0.100	0.500	0.100	0.077	0.923	1.000
1.000	0.500	0.100	0.500	0.500	0.048	0.952	1.000
1.000	0.500	0.100	0.500	1.000	0.032	0.968	1.000
1.000	0.500	0.100	1.000	0.100	0.083	0.917	1.000
1.000	0.500	0.100	1.000	0.500	0.063	0.938	1.000
1.000	0.500	0.100	1.000	1.000	0.048	0.952	1.000
1.000	0.500	0.500	0.050	0.100	0.143	0.857	1.000
1.000	0.500	0.500	0.050	0.500	0.043	0.957	1.000
1.000	0.500	0.500	0.050	1.000	0.023	0.977	1.000
1.000	0.500	0.500	0.100	0.100	0.200	0.800	1.000
1.000	0.500	0.500	0.100	0.500	0.077	0.923	1.000
1.000	0.500	0.500	0.100	1.000	0.043	0.957	1.000
1.000	0.500	0.500	0.250	0.100	0.263	0.737	1.000
1.000	0.500	0.500	0.250	1.000	0.091	0.909	1.000
1.000	0.500	0.500	0.500	0.100	0.294	0.706	1.000
1.000	0.500	0.500	0.500	0.500	0.200	0.800	1.000
1.000	0.500	0.500	0.500	1.000	0.143	0.857	1.000
1.000	0.500	0.500	1.000	0.100	0.313	0.688	1.000
1.000	0.500	0.500	1.000	0.500	0.250	0.750	1.000
1.000	0.500	0.500	1.000	1.000	0.200	0.800	1.000
1.000	0.500	1.000	0.050	0.100	0.250	0.750	1.000
1.000	0.500	1.000	0.050	0.500	0.083	0.917	1.000
1.000	0.500	1.000	0.050	1.000	0.045	0.955	1.000
1.000	0.500	1.000	0.100	0.100	0.333	0.667	1.000
1.000	0.500	1.000	0.100	0.500	0.143	0.857	1.000
1.000	0.500	1.000	0.100	1.000	0.083	0.917	1.000
1.000	0.500	1.000	0.250	0.100	0.417	0.583	1.000
1.000	0.500	1.000	0.250	0.500	0.250	0.750	1.000
1.000	0.500	1.000	0.250	1.000	0.167	0.833	1.000
1.000	0.500	1.000	0.500	0.100	0.455	0.545	1.000
1.000	0.500	1.000	0.500	0.500	0.333	0.667	1.000
1.000	0.500	1.000	0.500	1.000	0.250	0.750	1.000
1.000	0.500	1.000	1.000	0.100	0.476	0.524	1.000
1.000	0.500	1.000	1.000	0.500	0.400	0.600	1.000
1.000	0.500	1.000	1.000	1.000	0.333	0.667	1.000
1.000	1.000	0.100	0.050	0.100	0.032	0.968	1.000
1.000	1.000	0.100	0.050	0.500	0.009	0.991	1.000
1.000	1.000	0.100	0.050	1.000	0.005	0.995	1.000
1.000	1.000	0.100	0.100	0.100	0.048	0.952	1.000
1.000	1.000	0.100	0.100	0.500	0.016	0.984	1.000
1.000	1.000	0.100	0.100	1.000	0.009	0.991	1.000
1.000	1.000	0.100	0.250	0.100	0.067	0.933	1.000

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
1.000	1.000	0.100	0.250	0.500	0.032	0.968	1.000
1.000	1.000	0.100	0.250	1.000	0.020	0.980	1.000
1.000	1.000	0.100	0.500	0.100	0.077	0.923	1.000
1.000	1.000	0.100	0.500	0.500	0.048	0.952	1.000
1.000	1.000	0.100	0.500	1.000	0.032	0.968	1.000
1.000	1.000	0.100	1.000	0.100	0.083	1.875	1.958
1.000	1.000	0.100	1.000	0.500	0.063	0.0	0.063
1.000	1.000	0.500	0.050	0.100	0.143	0.857	1.000
1.000	1.000	0.500	0.050	0.500	0.043	0.957	1.000
1.000	1.000	0.500	0.050	1.000	0.023	0.977	1.000
1.000	1.000	0.500	0.100	0.100	0.200	0.800	1.000
1.000	1.000	0.500	0.100	0.500	0.077	0.923	1.000
1.000	1.000	0.500	0.100	1.000	0.043	0.957	1.000
1.000	1.000	0.500	0.250	0.100	0.263	0.737	1.000
1.000	1.000	0.500	0.250	0.500	0.143	0.857	1.000
1.000	1.000	0.500	0.250	1.000	0.091	0.909	1.000
1.000	1.000	0.500	0.500	0.100	0.294	0.706	1.000
1.000	1.000	0.500	0.500	0.500	0.200	0.800	1.000
1.000	1.000	0.500	0.500	1.000	0.143	0.857	1.000
1.000	1.000	0.500	1.000	0.100	0.313	1.813	2.125
1.000	1.000	1.000	0.050	0.100	0.250	0.750	1.000
1.000	1.000	1.000	0.050	0.500	0.083	0.917	1.000
1.000	1.000	1.000	0.050	1.000	0.045	0.955	1.000
1.000	1.000	1.000	0.100	0.100	0.333	0.667	1.000
1.000	1.000	1.000	0.100	0.500	0.143	0.857	1.000
1.000	1.000	1.000	0.100	1.000	0.083	0.917	1.000
1.000	1.000	1.000	0.250	0.100	0.417	0.583	1.000
1.000	1.000	1.000	0.250	0.500	0.250	0.750	1.000
1.000	1.000	1.000	0.250	1.000	0.167	0.833	1.000
1.000	1.000	1.000	0.500	0.100	0.455	0.545	1.000
1.000	1.000	1.000	0.500	0.500	0.333	0.667	1.000
1.000	1.000	1.000	0.500	1.000	0.250	0.750	1.000

PROBABILITY OF WINNING -- TANK VS HELICOPTER

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.100	0.100	0.100	0.500	0.100	0.972	0.472	1.444
0.100	0.100	0.100	0.500	0.500	0.801	0.506	1.308
0.100	0.100	0.100	0.500	1.000	0.694	0.528	1.222
0.100	0.100	0.100	1.000	0.100	1.169	0.474	1.643
0.100	0.100	0.100	1.000	0.500	1.010	0.490	1.500
0.100	0.100	0.100	1.000	1.000	0.889	0.502	1.391
0.100	0.100	0.500	0.100	0.100	0.515	0.250	0.765
0.100	0.100	0.500	0.100	0.500	0.496	0.314	0.810
0.100	0.100	0.500	0.100	1.000	0.481	0.365	0.846
0.100	0.100	0.500	1.000	0.100	1.026	0.054	1.081
0.100	0.100	0.500	1.000	0.500	1.010	0.066	1.076
0.100	0.100	0.500	1.000	1.000	0.992	0.078	1.070
0.100	0.100	1.000	0.100	0.100	0.511	0.207	0.719
0.100	0.100	1.000	0.100	0.500	0.505	0.245	0.750
0.100	0.100	1.000	0.100	1.000	0.499	0.282	0.781
0.100	0.100	1.000	0.500	0.100	1.096	0.089	1.185
0.100	0.100	1.000	0.500	0.500	0.877	0.057	0.934
0.100	0.100	1.000	0.500	1.000	0.870	0.068	0.938
0.100	0.500	0.100	0.500	0.100	0.676	0.769	1.444
0.100	0.500	0.100	0.500	0.500	0.494	0.814	1.308
0.100	0.500	0.100	0.500	1.000	0.380	0.843	1.222
0.100	0.500	0.100	1.000	0.100	0.857	0.786	1.643
0.100	0.500	0.100	1.000	0.500	0.691	0.809	1.500
0.100	0.500	0.100	1.000	1.000	0.565	0.826	1.391
0.100	0.500	0.500	0.100	0.100	0.358	0.407	0.765
0.100	0.500	0.500	0.100	0.500	0.306	0.504	0.810
0.100	0.500	0.500	0.100	1.000	0.263	0.583	0.846
0.100	0.500	0.500	1.000	0.100	0.925	0.156	1.081
0.100	0.500	0.500	1.000	0.500	0.889	0.187	1.076
0.100	0.500	0.500	1.000	1.000	0.850	0.221	1.070

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.100	0.500	1.000	0.100	0.100	0.375	0.344	0.719
0.100	0.500	1.000	0.100	0.500	0.346	0.404	0.750
0.100	0.500	1.000	0.100	1.000	0.317	0.463	0.780
0.100	0.500	1.000	0.500	0.100	0.796	0.134	0.931
0.100	0.500	1.000	0.500	0.500	0.772	0.162	0.934
0.100	0.500	1.000	0.500	1.000	0.745	0.193	0.938
0.100	1.000	0.100	0.500	0.100	0.616	0.828	1.444
0.100	1.000	0.100	0.500	0.500	0.436	0.872	1.308
0.100	1.000	0.100	0.500	1.000	0.323	0.899	1.222
0.100	1.000	0.100	1.000	0.100	0.789	0.854	1.643
0.100	1.000	0.100	1.000	0.500	0.624	0.876	1.500
0.100	1.000	0.100	1.000	1.000	0.498	0.893	1.391
0.100	1.000	0.500	0.100	0.100	0.326	0.439	0.765
0.100	1.000	0.500	0.100	0.500	0.270	0.540	0.810
0.100	1.000	0.500	0.100	1.000	0.224	0.622	0.846
0.100	1.000	0.500	1.000	0.100	0.879	0.202	1.081
0.100	1.000	0.500	1.000	0.500	0.836	0.240	1.076
0.100	1.000	0.500	1.000	1.000	0.789	0.282	1.070
0.100	1.000	1.000	0.100	0.100	0.345	0.374	0.719
0.100	1.000	1.000	0.100	0.500	0.312	0.438	0.750
0.100	1.000	1.000	0.100	1.000	0.279	0.501	0.780
0.100	1.000	1.000	0.500	0.100	0.757	0.174	0.931
0.100	1.000	1.000	0.500	0.500	0.726	0.208	0.934
0.100	1.000	1.000	0.500	1.000	0.691	0.247	0.938
0.500	0.100	0.100	0.500	0.100	0.713	0.731	1.444
0.500	0.100	0.100	0.500	0.500	0.622	0.686	1.308
0.500	0.100	0.100	0.500	1.000	0.565	0.657	1.222
0.500	0.100	0.100	1.000	0.100	1.018	0.882	1.900
0.500	0.100	0.100	1.000	0.500	0.857	0.786	1.643
0.500	0.100	0.100	1.000	1.000	0.751	0.722	1.474

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.500	0.100	0.500	0.100	0.500	0.385	0.425	0.810
0.500	0.100	0.500	0.100	1.000	0.391	0.455	0.846
0.500	0.100	0.500	1.000	0.100	1.026	0.167	1.192
0.500	0.100	0.500	1.000	0.500	0.990	0.177	1.167
0.500	0.100	0.500	1.000	1.000	0.957	0.186	1.143
0.500	0.100	1.000	0.100	0.500	0.375	0.344	0.719
0.500	0.100	1.000	0.100	1.000	0.386	0.371	0.757
0.500	0.100	1.000	0.500	0.500	0.742	0.133	0.875
0.500	0.100	1.000	0.500	1.000	0.744	0.145	0.889
0.500	0.500	0.100	0.500	0.100	0.269	1.176	1.444
0.500	0.500	0.100	0.500	0.500	0.212	1.096	1.308
0.500	0.500	0.100	0.500	1.000	0.176	1.046	1.222
0.500	0.500	0.100	1.000	0.100	0.444	1.456	1.900
0.500	0.500	0.100	1.000	0.500	0.349	1.294	1.643
0.500	0.500	0.100	1.000	1.000	0.287	1.187	1.474
0.500	0.500	0.500	0.100	0.500	0.131	0.679	0.810
0.500	0.500	0.500	0.100	1.000	0.122	0.724	0.846
0.500	0.500	0.500	1.000	0.100	0.718	0.474	1.192
0.500	0.500	0.500	1.000	0.500	0.667	0.500	1.167
0.500	0.500	0.500	1.000	1.000	0.619	0.524	1.143
0.500	0.500	1.000	0.100	0.500	0.153	0.566	0.719
0.500	0.500	1.000	0.100	1.000	0.147	0.610	0.757
0.500	0.500	1.000	0.500	1.000	0.481	0.407	0.889

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
0.500	1.000	0.100	0.500	0.100	0.185	1.259	1.444
0.500	1.000	0.100	0.500	0.500	0.138	1.170	1.308
0.500	1.000	0.100	0.500	1.000	0.108	1.114	1.222
0.500	1.000	0.100	1.000	0.100	0.323	1.577	1.900
0.500	1.000	0.100	1.000	0.500	0.244	1.399	1.643
0.500	1.000	0.100	1.000	1.000	0.191	1.282	1.474
0.500	1.000	0.500	0.100	0.500	0.085	0.724	0.810
0.500	1.000	0.500	0.100	1.000	0.075	0.772	0.846
0.500	1.000	0.500	1.000	0.100	0.583	0.609	1.192
0.500	1.000	0.500	1.000	0.500	0.528	0.639	1.167
0.500	1.000	0.500	1.000	1.000	0.476	0.667	1.143
0.500	1.000	1.000	0.100	0.500	0.107	0.612	0.719
0.500	1.000	1.000	0.100	1.000	0.098	0.658	0.757
0.500	1.000	1.000	0.500	1.000	0.370	0.519	0.889
1.000	0.100	0.100	0.500	0.100	0.593	0.704	1.296
1.000	0.100	0.100	0.500	0.500	0.552	0.676	1.229
1.000	0.100	0.100	0.500	1.000	0.522	0.656	1.178
1.000	0.100	0.100	1.000	0.100	0.818	0.825	1.643
1.000	0.100	0.100	1.000	0.500	0.737	0.763	1.500
1.000	0.100	0.100	1.000	1.000	0.676	0.715	1.391
1.000	0.100	0.500	0.100	1.000	0.385	0.484	0.869
1.000	0.100	0.500	1.000	0.100	0.979	0.213	1.192
1.000	0.100	0.500	1.000	0.500	0.949	0.217	1.167
1.000	0.100	0.500	1.000	1.000	0.922	0.221	1.143
1.000	0.100	1.000	0.100	1.000	0.379	0.401	0.780

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
1.000	0.100	1.000	0.500	1.000	0.717	0.172	0.889
1.000	0.500	0.100	0.500	0.100	0.173	1.123	1.296
1.000	0.500	0.100	0.500	0.500	0.152	1.076	1.229
1.000	0.500	0.100	0.500	1.000	0.137	1.041	1.178
1.000	0.500	0.100	1.000	0.100	0.286	1.357	1.643
1.000	0.500	0.100	1.000	1.000	0.217	1.174	1.391
1.000	0.500	0.500	0.100	1.000	0.101	0.768	0.869
1.000	0.500	1.000	0.100	1.000	0.122	0.659	0.780
1.000	1.000	0.100	0.500	0.100	0.098	1.199	1.296
1.000	1.000	0.100	0.500	0.500	0.082	1.146	1.229
1.000	1.000	0.100	0.500	1.000	0.071	1.107	1.178
1.000	1.000	0.100	1.000	0.100	0.175	1.468	1.643
1.000	1.000	0.100	1.000	0.500	0.146	1.354	1.500
1.000	1.000	0.100	1.000	1.000	0.125	1.267	1.391
1.000	1.000	0.500	0.100	1.000	0.052	0.817	0.869

TK DET	TK KILL	HEL DET	HEL KIL	HEL HID	TK WIN	HEL WIN	CHECK
1.000	1.000	0.500	1.000	0.100	0.423	0.769	1.192
1.000	1.000	0.500	1.000	0.500	0.389	0.778	1.167
1.000	1.000	0.500	1.000	1.000	0.357	0.786	1.143
1.000	1.000	1.000	0.100	1.000	0.070	0.711	0.780

COMPUTER PROGRAM

OF

GENERAL MODEL

```
      DIMENSION A1(3),A2(3), B1(3), B2(5), C(3),A3(3)
      DIMENSION B3(5)
      DATA A1/.1,.5,1.0/
      DATA A2/.1,.5,1.0/
      DATA A3/.1,.5,1.0/
      DATA B1/.1,.5,1.0/
      DATA B2/.05,.1,.25,.5,1.0/
      DATA B3/.05,.1,.25,.5,1.0/
      WRITE (6,8400)
 8400 FORMAT (1H1,40X,'COMPUTER OUTPUT//GENERAL MODEL',/////)
      WRITE (6,8500)
 8500 FORMAT (1X, '      PROBABILITY OF WINNING --',
     1'TANK VS HELICOPTER',///)
      WRITE (6,8900)
 8900 FORMAT(1X,22X,'HELDET  HELKILL  TK DET  TK KILL HEL',
     1' WIN TK WIN   CHECK',/)
      WRITE (6,9000)
 9000 FORMAT (1X,20X,'      A1       A2       B1       B2',/)
      DO 50 I=1,3
      DO 40 J=1,3
      DO 30 K=1,3
      DO 20 L=1,5
      JJ = J
      LL=L
      IF (L.EQ.1) GO TO 5
      GO TO 1
    5 WRITE (6,9005)
 9005 FORMAT (1X)
    1 CONTINUE
 0100 TK = (B1(K)*B2(L))/((A1(I)+B1(K)) * (A1(I)+B2(L)))
      HEL = (A1(I)*A2(J)) / ((A1(I)+B1(K)) * (A2(J)
     1  + B1(K)))
      COM = ((A1(I)+A2(J)+B1(K) + B2(L))*A1(I) * B1(K))/
     1((A1(I) + B1(K))*(A3(JJ)+B3(LL))*(A1(I)+B2(L))*(A2(J)
     2  + B1(K)))
      HWIN = HEL + COM*A3(JJ)
      TWIN = TK + COM *B3(LL)
      CHEK = HWIN + TWIN
      WRITE (6,9510) A1(I),A2(J),B1(K),B2(L),HWIN,TWIN,CHEK
 9510 FORMAT (' ',20X,7F8.3)
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      STOP
      END
```

COMPUTER PROGRAM

OF

MODEL I

```
      DIMENSION A1(3),A2(3), B1(3), B2(5), C(3),A3(3 ),B3(3)
      DATA A1/.1,.5,1.0/
      DATA A2/.1,.5,1.0/
      DATA B1/.1,.5,1.0/
      DATA B2/.05,.1,.25,.5,1.0/
      DATA  C/.1,.5,1.0/
      WRITE (6,8400)
 8400 FORMAT (1H1,40X,'COMPUTER OUTPUT -- MODEL I',////)
      WRITE (6,8500)
 8500 FORMAT (1X, 27X,    'PROBABILITY OF WINNING -- ',
     1'TANK VS HELICOPTER',///)
      WRITE (6,9000)
 9000 FORMAT (1X,22X,'TK DET  TK KILL HEL DET HEL KIL HEL',
     1' HID  TK WIN  HEL WIN  CHECK',/)
      WRITE (6,9100)
 9100 FORMAT (1X,24X,'A1       A2       B1       B2       C',/)
      DO 50 I=1,3
      DO 40 J=1,3
      DO 30 K=1,3
      DO  20 L=1,5
      DO 10 M=1,3
      IF (M.EQ.1) GO TO 5
      GO TO 1
    5 WRITE (6,9005)
 9005 FORMAT (1X)
    1 CONTINUE
      S=((A1(I)+B1(K)+B2(L)+C(M))**2)-4.0*(A1(I)*B2(L)+A1(I)
     1 *C(M) + B1(K)*B2(L))
      IF (S.LT.0.0) GO TO 10
      SSQT = SQRT (S)
      S1 = (SSQT/2.0) **2
      B=-(A1(I)+B1(K)+B2(L)+C(M))
      R=(B2(L)+C(M)-A1(I)-B1(K))/2.0
      T=((2.0*A2(J))-B2(L)-C(M)-A1(I)-B1(K))/2.0
      DEN1 = S1 - (T**2)
      IF (DEN1 .EQ. 0.0) GO TO 10
      DEN2 = S1 - (R**2)
      IF (DEN2 .EQ. 0.0) GO TO 10
      W = (A1(I) * A2(J))/ DEN1
      V = 1.0 / (S1 - ((B/2.0)**2))
      P4 = -B1(K) * B2(L) * V
      P5 =W*V*((R*T)-S1+(B/2.0)*(R-T))+(W/A2(J))*(A2(J)
     1 -B2(L) - C(M))
      CHEK = P4 + P5
      WRITE (6,9010) A1(I),A2(J),B1(K),B2(L),C(M),P4,P5,CHEK
 9010 FORMAT (1X,20X,8F8.3)
   10 CONTINUE
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      STOP
      END
```

COMPUTER PROGRAM

OF

MODEL II

```
      DIMENSION A1(3),A2(3), B1(3), B2(3), C(3),A3(3 ),B3(3)
      DATA A1/.1,.5,1.0/,A2/.1,.5,1.0/
      DATA B1/.1,.5,1.0/,B2/.1,.5,1.0/,C/.1,.5,1.0/
      WRITE (6,8400)
 8400 FORMAT (1H1,40X,'COMPUTER OUTPUT -- MODEL II',////)
      WRITE (6,8500)
 8500 FORMAT (1X, 27X,   'PROBABILITY OF WINNING -- ',
     1' TANK VS HELICOPTER',///)
      WRITE (6,9000)
 9000 FORMAT (1X,'   TK DET   TK KILL  HEL DET  HEL KILL',
     1' HEL HID  TK WIN  HEL WIN  CHECK',/)
      DO 50 I=1,3
      DO 40 J=1,3
      DO 30 K=1,3
      DO 20 L=1,3
      DO 10 M=1,3
      IF (M.EQ.1) GO TO 5
      GO TO 1
    5 WRITE (6,9005)
 9005 FORMAT (1X)
    1 CONTINUE
      S = (2.0*A1(I)+B1(K)+B2(L)+C(M))**2-(4.0*(A1(I)**2
     1+A1(I)*B1(K) +A1(I)*B2(K)+A1(I)*C(M)+B1(K)*B2(L)))
      IF (S.LT.0.0) GO TO 10
      SSQT = SQRT(S)
      S1 =  (SSQT/2.0) **2
      R = (B2(L) + C(M) - B1(K)) / 2.0
      B = -2.0*A1(I) - B1(K) - B2(L) - C(M)
      BD = B/2.0
      H = 2.0*A2(J) - 2.0*A1(I) + B2(L) - B1(K) - C(M)
      HD = H/2.0
      BDEN = S1 - BD**2
      IF (BDEN .EQ.0.0) GO TO 10
      HDEN = S1 - HD**2
      IF (HDEN .EQ. 0.0) GO TO 10
      AD = 1.0/(A2(J) + B2(L))
      AC = 1.0 / (A2(J) + B1(K))
      DEN3 = B2(L) - B1(K)
      IF (DEN3 .EQ. 0.0) GO TO 10
      T =(-2.0*A1(I) + 2.0*A2(J) + B1(K)-B2(L)- C(M)) / 2.0
      DEN1 = S1-(T**2)
      IF (DEN1 .EQ. 0.0)  GO TO 10
      Y =(B1(K)*B2(L) *A1(I)) / DEN1
      DEN2 = S1 - (R**2)
      IF (DEN2 .EQ. 0.0)  GO TO 10
      W = (A1(I) * A2(J))/ DEN1
      V = 1.0 / (S1 - ((B/2.0)**2))
      V2= 1.0 / (S1 - ((H/2.0)**2))
      U1 = A1(I) * B1(K) * B2(L)  / DEN2
      U2 = A1(I) * B1(K) * A2(J)  / DEN2
      Z = A1(I) *A2(J)  * B1(K) / DEN1
      P5A =-B1(K) * B2(L) * V
      P5B =Y*V2*(S1-R*T) * (BD*V+HD*V-AD)
      P5C = U1 * V2    *(S1-R**2) * (BD*V+HD*V - AD)
      P5D =Y*V2*(R-T)*(-S1*V-HD*BD*V+HD*AD)
      P5E = Y*(AC-AD) *   (T-R)/DEN3
      P5 = P5A + P5B + P5C + P5D + P5E
      P6A=W*(R*T*V-S1*V+(R-T)*V*B/2.0+(T-R)/(A2(J)+B1(K)))
      P6B =Z*V2*(S1-R*T) * (BD*V+HD*V-AD)
      P6C = U2 * V2    *(S1-R**2) * (BD*V+HD*V - AD)
      P6D= Z*V2*(R-T)*(-S1*V-HD*BD*V+HD*AD)
```

```
      P6E = Z*(AC-AD) *  (T-R)/DEN3
      P6=P6A + P6B + P6C + P6D + P6E
      CHEK = P5 + P6
      WRITE (6,9010) A1(I),A2(J),B1(K),B2(L),C(M),P5,P6,CHEK
 9010 FORMAT (1X,20X,8F8.3)
   10 CONTINUE
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      STOP
      END
```

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13. ABSTRACT

The purpose of this thesis is to mathematically model a duel between the armed helicopter and the tank. In addition to providing a parametric analysis of B. O. Koopman's classical Detection-Destruction Duel, two additional models were constructed and analyzed. All three models stem from stochastic versions of Lanchester's Equations but require that a unit first be detected before it is destroyed. The later two models are extensions of Koopman's model but provide for the unique capability of the helicopter to rapidly maneuver behind masking terrain, thus transitioning from the detected state back to the undetected state. With further refinement, these models may prove to be a viable alternative to the current method of computer simulation.

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